Volume No: 1 Issue 2 June 2017

International Multidisciplinary Innovative Research Journal
-An International refereed e-journal - Science Issue
ISSN: 2456-4613
IMIRJ, 1(2), 2017 [1-6]

FUZZY $\hat{\Omega}$ -CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

SANGEETHA.M[1] AND ANBUCHELVI.M[2]
[1]M.Phil. Scholar
[2]Associate Professor
Department of Mathematics
V.V.Vanniaperumal College for Women
Virudhunagar. TamilNadu, India.

ABSTRACT

This paper aims to fuzzify the concept of $\hat{\Omega}$ - closed set in fuzzy topological space. Some basic properties have been derived. By giving suitable examples, it is shown that this new class lies between the class of fuzzy δ -closed sets and fuzzy δ -generalised closed sets.

Keywords : Fuzzy generalized set, Fuzzy $\hat{\Omega}$ -closed set, Fuzzy set, Fuzzy topology.
AMS Mathematics Subject Classification (2010): 54A40 , 03E72.

1 INTRODUCTION

In 1965, Zadeh [Zadeh, 1965] introduced the concept of fuzzy set by defining it in terms of mapping from a set into the unit interval on real line. In 1968, the study of fuzzy topology was investigated by Chang [Chang, 1968]. The theory of fuzzy topological space was subsequently developed by several authors. In 1998, H.Maki et al [Maki et al., 1998] introduced the concept of generalised closed sets in a fuzzy topological space. Recently,

$\hat{\Omega}$ -closed set has been introduced by M.Lellis Thivagar et al. In this paper, we extend the definition of $\hat{\Omega}$-closed set into fuzzy topological space in the name fuzzy $\hat{\Omega}$ -closed set and study its basic properties.

2 PRELIMINARIES

Now we recall some of the basic definitions in fuzzy topology.

Definition 2.1: [Zadeh, 1965] Let X be a non-empty set . A **fuzzy set** A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A, for each $x \in X$. It is clear that A is completely determined by the set of tuples $A=\{(x, \mu_A(x)) : x \in X\}$.

Definition 2.2: [Zadeh, 1965] Let $A=\{(x, \mu_A(x)) : x \in X\}$ and $B = \{(x, \mu_A(x)) : x \in X\}$ be two fuzzy sets in X. Then their union $A \cup B$, intersection $A \cap B$ and complement A^C are also fuzzy sets with the membership functions defined as follows:

- $\mu_{A \vee B}(x)=\max\{\mu_A(x), \mu_B(x)\}$, $\forall x \in X$.
- $\mu_{A \wedge B}(x)=\min\{\mu_A(x), \mu_B(x)\}$, $\forall x \in X$.
- $\mu_{A^c}(x) = 1-\mu_A(x), \forall x \in X$.

Further,

(a) $A \subseteq B$ iff $\mu_A(x) \le \mu_B(x), \forall x \in X$.

(b) $A = B$ iff $\mu_A(x) = \mu_B(x), \forall x \in X$.

Definition 2.3: [Palaniappan, 2002] A family $\tau \subseteq I^X$ of fuzzy sets is called a **fuzzy topology** for X if it satisfies the following three axioms:

1. $0, 1 \in \tau$.
2. $\forall A, B \in \tau \Rightarrow A \wedge B \in \tau$.
3. $\forall (A_j)_{j \in J} \in \tau \Rightarrow \vee_{j \in J} A_j \in \tau$.

The pair (X, τ) is called a fuzzy topological space (or) fts, for short. The elemets of τ are called fuzzy open sets. A fuzzy set K is called fuzzy closed if $K^C \in \tau$.

Definition 2.4: [Palaniappan, 2002] The closure $\overline{A}$ and the interior A^0 of a fuzzy set A of X are defined as

$$\overline{A} = \inf\{K : A \le K, K^C \in \tau\}$$

$$A^0 = \sup\{O : O \le A, O \in \tau\}$$

respectively.

Definition 2.5: [Azad, 1981] A fuzzy set A of a fuzzy topological space(X, τ) is said to be **fuzzy semi-open**, if $A \le (cl(int(A))$ and the complement of fuzzy semi open set is fuzzy semi closed set in X .

Definition 2.6: [Azad, 1981] A fuzzy set A of a fuzzy topological space (X, τ) is said to be **fuzzy regular closed**, if $A = cl(int(A))$.

Definition 2.7: [Ganguly and saha, 1988] A fuzzy set A of a fuzzy topological space (X, τ) is said to be a **fuzzy δ-closed set**, if $A = cl_\delta(A)$, where $cl_\delta(A) = \wedge\{F \subseteq I^X / A \leq F, F = cl(int(F))\}$.

Definition 2.8: [Sudha et al., 2011] A fuzzy set A of a fuzzy topological space (X, τ) is said to be **fuzzy ω-closed set,** if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is fuzzy semi-open in X.

Definition 2.9: [Balasubramanian and Sundaram, 1997] A fuzzy set A of a fuzzy topological space (X, τ) is said to be **fuzzy generalized closed set**(in short, fg-closed), if $cl(A) \subseteq U$, whenever $A \subseteq U$ and is fuzzy open in X.

Definition 2.10: A fuzzy set A of a fuzzy topological space (X, τ) is said to be **fuzzy δg -closed set,** if $cl_\delta(A) \subseteq U$, whenever $A \subseteq U$ and U is fuzzy open in X.

Results 2.11:

- [Allam and Zahran, 1992] Every fuzzy δ-closed set is fuzzy closed set.
- [Maki et al., 1998] Every fuzzy δg closed set is fuzzy g-closed set.
- [Sudha et al., 2011] Every fuzzy closed set is fuzzy ω-closed set.
- [Sudha et al., 2011] Every fuzzy ω-closed set is fuzzy g-closed set.

3 FUZZY $\hat{\Omega}$-CLOSED SET

Definiition 3.1: A fuzzy set A of a fuzzy topological space (X, τ) is said to be **fuzzy $\hat{\Omega}$ -closed set,** if $cl_\delta(A) \subseteq U$, whenever $A \subseteq U$ and U is fuzzy semi- open in X. It is denoted by $f\hat{\Omega}-$closed set. Its complement is called fuzzy $\hat{\Omega}$-open set in X.

Example 3.2: Consider $X = \{a, b, c\}$ and $\tau = \{0, 1, A, B, C, D\}$. The fuzzy sets A,B,C, and D are given in the following table.

fs(X)	a	b	c
A	0.4	0.2	0.5
B	0.6	0.1	0.3
C	0.6	0.2	0.5
D	0.4	0.1	0.3

f$\hat{\Omega}$-closed sets are $\{0, 1, A', B', \ldots\}$

Proposition 3.3: Every fuzzy δ-closed set is f$\hat{\Omega}$-closed set.

Proof Assume that A is fuzzy δ-closed set in a fuzzy topological space (X , τ). Let $A \subseteq U$ where U is fuzzy semi-open. Since A is fuzzy δ-closed, by [5] $A = cl_{\delta}(A)$. So, A is f$\hat{\Omega}$-closed set.

Remark 3.4: The converse of the above statement is not true in general as shown by the example.

Example 3.5: Consider $X = \{a, b, c\}$ and $\tau = \{0, 1, A, B, C, D\}$. The fuzzy sets A,B,C, and D are given in the following table.

fs(X)	a	b	c
A	0.5	0.3	0.4
B	0.1	0.2	0.6
C	0.5	0.3	0.6
D	0.1	0.2	0.4

B' is a f$\hat{\Omega}$-closed set but not a fuzzy δ-closed set.

Proposition 3.6: Every f$\hat{\Omega}$-closed set is fuzzy δg-closed set.

Proof Assume that A is any f$\hat{\Omega}$-closed set in a fuzzy topological space (X, τ).Let $A \subseteq U$ where U is fuzzy open. By [5], U is fuzzy semi-open. By hypothesis, $cl_{\delta}(A) \subseteq U$. So, A is fuzzy δg-closed set.

Remark 3.7: The converse of the above statement is not true in general as shown by the following example.

Example3.8:

Consider $X = \{a, b, c\}$ and $\tau = \{0, 1, A, B, C\}$.

The fuzzy sets A,B, and C are given in the following table.

fs(X)	a	b	c
A	0.2	0.1	0.4
B	0.4	0.2	0.5
C	0.5	0.3	0.5
D	0.5	0.4	0.6

D ‘is a fuzzy δg-closed set but not a f$\hat{\Omega}$-closed set.

Proposition 3.9: Every f$\hat{\Omega}$-closed set is a fuzzy ω-closed set.

Proof Assume that A is a f$\hat{\Omega}$-closed set in a fuzzy topological space (X, τ). Let $A \subseteq U$ where U is fuzzy semi-open. By hypothesis, $cl_{\delta}(A) \subseteq U$. By [9], $cl(A) \subseteq U$. So, A is fuzzy ω-closed set.

Remark 3.10: The following example shows that the converse of the above result is not always true.

Example 3.11: Let $X = \{a, b, c\}$ and $\tau = \{0, 1, A, B, C, D\}$. The fuzzy sets A,B, and C are given in the following table.

fs(X)	*a*	*b*	*c*
A	0.1	0.3	0.4
B	0.5	0.6	0.7
C	0.6	0.7	0.8
D	0.7	0.8	0.9

C' is fuzzy ω -closed set but not $f\hat{\Omega}$-closed set.

Remark 3.12: From the following two examples, it is known that $f\hat{\Omega}$-closed set is independent of fuzzy closed set.

Example 3.13: Let $X = \{a,b,c\}$ and $\tau = \{0,1,A,B,C\}$. The fuzzy sets A,B,C, and D are given in the following table.

fs(X)	*a*	*b*	*c*
A	0.2	0.1	0.4
B	0.4	0.2	0.5
C	0.5	0.3	0.5
D	0.5	0.4	0.6

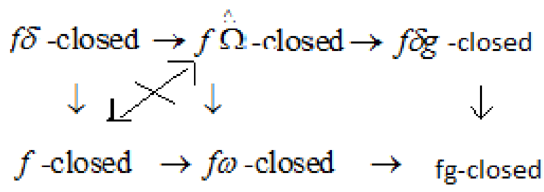
D is a $f\hat{\Omega}$ -closed set but not a fuzzy closed set.

Example3.14:Let $X = \{a,b,c\}$ and $\tau = \{0,1,A,B\}$. The fuzzy sets A and B are given in the following table.

fs(X)	*a*	*b*	*c*
A	0.1	0.3	0.4
B	0.5	0.6	0.7
C	0.6	0.7	0.8
D	0.7	0.8	0.9

B' is fuzzy closed but not $f\hat{\Omega}$-closed set.

Remark 3.15: From the above discussions and from [1, 7, 11] the following diagram has been captured.



CONCLUSION

The class of $\hat{\Omega}$ - closed sets has been extended to fuzzy topologcal space. It has been shown that this class properly lies between the class of fuzzy δ -closed sets and fuzzy δg -closed sets.

REFERENCES

Allam A.A. and zahran A.A, (1992), *Fuzzy sets and systems*, **50**, 103-112.

Azad K.K, (1981), *On fuzzy semi continuity, Fuzzy almost continuity and weakly continuity* , J.Math. Anal. Appl . **82**, 14-32.

Balasubramanian G. and Sundaram P, (1997), *On some generalizations of fuzzy continuous function*, **86**, 92-100.

Chang C.L., (1968), *Fuzzy topological space,* J.Math. Anal. Appl . **24**, pp.182-190.

Ganguly S. and Saha S, (1988), *A note on δ -continuous and δ -connected sets in fuzzy set theory,* Simon Stevin, **62,** 127-141.

Lellis Tivagar M., Anbuchelvi M, (2015), *Note on $\hat{\Omega}$ -closed sets in fuzzy topological spaces*, Mathematics Theory and Modeling, **5**.

Maki H, Fukutake T, Kojima M. and Huruda H, (1998), *Generalized closed sets fuzzy topological spaces theory and its applications,* 23-36.

Manoj Mishra, Thakur S.S. (2012), *Fuzzy ω -continuous mappings,* **2,** Issue 12.

Mukherjee M.N. and Sinha S.P, (1990), *On some near-fuzzy functions between fuzzy topological space,* Fuzzy sets and system, **34**(2)), 245-254.

Palaniappan N, (2002), *Fuzzy topology*, Narosa Publications.

Sudha M, Roja E.and Uma M.K., (2011), *Slightly fuzzy w-continuous mappings,* Int.J.of Math. Anal, **5**, No.16, 779-787.

Zadeh L.A, (1965), *Fuzzy Sets*, Information and Control, **8**, 338-353.