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FUZZY SOFT KERNEL IN FUZZY SOFT TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to introduce and discuss the concept of fuzzy soft kernel and fuzzy soft R_0 space for fuzzy soft topological space.

Keywords: Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft interior and Fuzzy soft closure.
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1 INTRODUCTION

In 1965, Zadeh [Zadeh, 1965] introduced the fundamental concept of fuzzy sets, which formed the backbone of fuzzy mathematics. Fuzzy topological space has been introduced by Chang [Chang, 1968] in 1968. In 1999, Molodstov [Molodstov, 1995] introduced the soft set theory, which is completely a new approach for modeling uncertainty. The concept of soft sets has been carried out in many fields such as smoothness of functions, game theory, Riemann integrations, and theory of probability. Shabir and Naz [Shabir and Naz, 2011] presented soft topological space and defined some concepts based on soft sets. In 2001, Maji [Maji et al., 2001] introduced the fuzzy soft set which is a combination of fuzzy set and soft set and they studied their properties. Tanay and Kandemir [Tanay and Kandemir, 2011] initially introduced the concept of fuzzy soft topological space using fuzzy soft sets and studied the basic notions by Chang's [Chang, 1968] fuzzy topology. In 2013, Cigdem and Gunduz et al. discussed some basic but important properties of fuzzy soft topological space such as fuzzy soft interior, fuzzy soft closure, fuzzy soft continuous mapping, fuzzy soft open mapping and fuzzy soft closed mapping. In this paper, we extend the concept of kernel for fuzzy soft set and fuzzy soft point. And we introduce the concept of fuzzy soft R_0 space and their properties. Also, we study the relation between fuzzy soft kernel and fuzzy soft R_0 space in fuzzy soft topological space.

2 PRELIMINARIES

Definition 2.1 [Palaniappan , 2002] A family $\tau \subseteq I^X$ of fuzzy sets is called a ***fuzzy topology*** for if it satisfies the following three axioms:

1. $\bar{0}, \bar{1} \in \tau$.
2. $\forall\ A, B \in \tau \Rightarrow A \wedge B \in \tau$.
3. $\forall\ (A_j)_{j \in J} \in \tau \Rightarrow \bigvee_{j \in J} (A_j) \in \tau$.

The pair (X, τ) is called a fuzzy topological space.

Definition 2.2 [Molodstov , 1995] Let τ be the collection of soft sets over X, then τ is said to be a **soft topology** on X if

1. $\phi, \overline{X}$ belong to τ.
2. The union of any number of soft sets in τ belong to τ.
3. The intersection of any two soft sets in τ belong to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.3 [Maji et al., 2001] Let I^X denote the set of all fuzzy sets on X and $A \subset E$. A pair (f, A) is called a **fuzzy soft set** over X, where f is a mapping from A into I^X.

Definition 2.4 [Mahanta and Das , 2012] The fuzzy soft set $f_A \in \mathrm{F}(X, E)$ is called **fuzzy soft point**, if there exist $x \in X$ and $e \in E$ such that $\mu_{f_A}^e(x) = \alpha$ $(0 \le \alpha \le 1)$ and $\mu_{f_A}^e(y) = 0$, $\forall\ y \in X - x$. This fuzzy soft point is denoted by x_α^e or f_e.

Theorem 2.5 [Mahanta and Das, 2012] Union of all fuzzy soft points of a fuzzy soft set is equal to fuzzy soft set.

Definition 2.6 [Maji et al., 2001] For two fuzzy soft sets (f, A) and (g, B) over a common universe X, we say that (f, A) is ***fuzzy soft subset*** of (g, B) if

1. $A \subset B$.
2. For each $a \in A, f_a \le g_a$; that is, f_a is fuzzy subset of g_a.

It is denoted by $(f, A) \subseteq (g, B)$

Definition 2.7 [Maji et al., 2001] The **union** of two fuzzy soft sets (f, A) and (g, B) over a common universe X is a fuzzy soft set (h, C), where $C = A \cup B$, and for all $c \in C$,

$h_c = f_c$ if $c \in A - B$, $h_c = g_c$ if $c \in B - A$, $h_c = f_c \vee g_c$ if $c \in A \cap B$. This relationship is denoted by $(f, A) \cup (g, B) = (h, C)$.

Definition 2.8 [Maji et al., 2001] The **intersection** of two fuzzy soft sets (f, A) and (g, B) over a common universe X is a fuzzy soft set (h, C), where $C = A \cap B$, and for all $c \in C$ $h_c = f_c \wedge g_c$. This relationship is denoted by $(f, A) \cap (g, B) = (h, C)$.

Definition 2.9 [Tanay and Kandemir , 2011] A **fuzzy soft topological space** is a pair (X, τ) where X is a nonempty set and τ is a family of fuzzy soft sets over X satisfying the following properties:

1. $\overline{\phi_E}$, $\overline{1_E}$ ∈ τ.
2. If $f_A, g_B \in \tau$, then $f_A \cap g_B \in \tau$.
3. If $(f_A)_i \in$ τ, for all $i \in J$, then $\bigcup_{i \in J} (f_A)_i \in \tau$.

τ is called a topology of fuzzy soft sets on X. Every member of τ is called fuzzy soft open set. A fuzzy soft set is called τ - closed iff its complement is τ - open.

Definition 2.10 [Pazar Varol and Aygun , 2012] Let (X, τ) be a fuzzy soft topological space and $f_A \in F(X, E)$. The **fuzzy soft closure** of f_A, denoted by $\overline{f_A}$, is the intersection of all fuzzy soft closed supersets of f_A.

Definition 2.11 [Pazar Varol and Aygun , 2012] Let (X, τ) be a fuzzy soft topological space and $f_A \in F(X, E)$. The **fuzzy soft interior** of f_A, denoted by f_A^o, is the union of all fuzzy soft open subsets of f_A.

3 FUZZY SOFT KERNELS

In this section, we extend the definition of kernel on fuzzy soft set theory and discuss some of its basic properties. Throughout this paper, X denotes an initial universe and E is set of parameters, $FSO(X, \tau, E)$ be the family of all fuzzy soft open sets over X via parameters in E and $F(x, e)$ is a fuzzy soft point in (X, τ, E).

Definition 3.1 Let (X, τ, E) be a fuzzy soft topological space. Let (F, E) be a fuzzy soft set over X. Then, a **fuzzy soft kernel** of (F, E), denoted by $fs\ker((F, E))$, is defined to be the set

$$fs\ker((F,E)) = \bigcap \{(G,E) : (F,E) \subseteq (G,E), (G,E) \in \tau\}$$

Always,

- $(F, E) \subseteq fs\ker((F, E))$
- $fs\ker(\phi_E) = \phi_E$
- $fs\ker(1_E) = 1_E$

Example 3.2 Let $X = \{x_1, x_2, x_3\}, E = \{e_1, e_2\}$,

$\tau = \{\phi_E, (F_1, E), (F_2, E), (F_3, E), (F_4, E), 1_E\}$

where $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ are fuzzy soft sets over X and defined as follows.

(F_1, E)	x_1	x_2	x_3
e_1	0.2	0.5	0.1
e_2	0.4	0	0.4

(F_2,E)	x_1	x_2	x_3
e_1	0.2	0.3	0.5
e_2	0.1	0.4	0

(F_3,E)	x_1	x_2	x_3
e_1	0.2	0.5	0.5
e_2	0.4	0.4	0.4

(F_4,E)	x_1	x_2	x_3
e_1	0.2	0.3	0.1
e_2	0.1	0	0

Here,

$$fs\ker((F_1,E)) = (F_1,E)$$

$$fs\ker((F_2,E)) = (F_2,E)$$

$$fs\ker((F_3,E)) = (F_3,E)$$

$$fs\ker((F_4,E)) = (F_4,E)$$

(F_5,E)	x_1	x_2	x_3
e_1	0.2	0.5	0.3
e_2	0.2	0	0.4

is fuzzy soft set over X.

Then, $fs\ker((F_5,E)) = (F_3,E)$

Definition 3.3 Let $F(x,e)$ be a fuzzy soft point of a fuzzy soft topological space (X,τ,E) Then, fuzzy soft kernel of $F(x,e)$ is defined to be the set

$$fs\ker(F(x,e)) = \bigcap\{(F,E) : F(x,e) \in (F,E), (F,E) \in \tau\}$$

Example 3.4 Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\phi_E, (F_1,E), 1_E\}$ where (F_1,E) is a fuzzy soft set over X and is defined as follows:

(F_1,E)	x_1	x_2
e_1	0.2	0.7
e_2	0.6	0.4

A fuzzy soft points in (F_1,E) are,

$F_1(x_1,e_1)$	x_1	x_2
e_1	0.2	0
e_2	0	0

$F_1(x_2,e_1)$	x_1	x_2
e_1	0	0.7
e_2	0	0

$F_1(x_1,e_2)$	x_1	x_2
e_1	0	0
e_2	0.6	0

$F_1(x_2,e_2)$	x_1	x_2
e_1	0	0
e_2	0	0.4

Here,

$$fs\ker(F(x_1,e_1)) = (F_1,E)$$

$$fs\ker(F(x_2,e_1)) = (F_1,E)$$

$$fs\ker(F(x_1,e_2)) = (F_1,E)$$

$$fs\ker(F(x_2,e_2)) = (F_1,E)$$

Now,

$F(x,e)$	x_1	x_2
e_1	0	0
e_2	0	0.5

is fuzzy soft point in (X,τ,E). Then, $fs\ker(F(x,e)) = 1$.

Theorem 3.5 Let (X,τ,E) be a fuzzy soft topological space. If (F,E) is a fuzzy soft open set, then $fs\ker((F,E))=(F,E)$.

Proof: Let (F,E) be a fuzzy soft open set in a fuzzy soft topological space (X,τ,E). By the definition of fuzzy soft kernel, $(F,E)\subseteq fs\ker((F,E))$ ..... (1)
Since (F,E) is fuzzy soft open, $fs\ker((F,E))\subseteq(F,E)$ ..... (2)
From (1) and (2), we have $fs\ker((F,E))=(F,E)$.

Theorem 3.6 Let (X,τ,E) be a fuzzy soft topological space. If $(F,E)\subseteq(G,E)$, then $fs\ker((F,E))\subseteq fs\ker((G,E))$ for fuzzy soft open sets (F,E) and (G,E).

Proof: Let (F,E) and (G,E) be any two fuzzy soft open sets such that $(F,E)\subseteq(G,E)$. Since (F,E) and (G,E) are fuzzy soft open sets, $fs\ker((F,E))=(F,E)$ and $fs\ker((G,E))=(G,E)$. Hence, $fs\ker((F,E))\subseteq fs\ker((G,E))$.

Theorem 3.7 Let (X,τ,E) be a fuzzy soft topological space. Let (F,E) be any fuzzy soft set over X. Then, $fs\ker(fs\ker((F,E)))=fs\ker((F,E))$.

Proof: Let (F,E) be any fuzzy soft set over X. By the definition of fuzzy soft kernel, $fs\ker((F,E))$ is a fuzzy soft open set. Therefore,

$$fs\ker(fs\ker((F,E)))=fs\ker((F,E)).$$

Definition 3.8 A fuzzy soft topological space (X,τ,E) is called fuzzy soft R_o - space if for each fuzzy soft open set (F,E), $\overline{(F(x,e))}\subseteq (F,E)$, for all $F(x.e)\in(F,E)$.

Example 3.9 Let $X=\{x_1,x_2\}$, $E=\{e_1,e_2\}$ and $\tau=\{\phi_E,(F_1,E),(F_2,E),(F_3,E),(F_4,e),1_E\}$ where $(F_1,E),(F_2,E),(F_3,E),(F_4,E)$ are fuzzy soft sets over X and defined as follows:

(F_1,E)	x_1	x_2
e_1	0.6	0.2
e_2	0.1	0.3

(F_2,E)	x_1	x_2
e_1	0.4	0.8
e_2	0.9	0.7

(F_3,E)	x_1	x_2
e_1	0.6	0.8
e_2	0.9	0.7

(F_4,E)	x_1	x_2
e_1	0.4	0.2
e_2	0.1	0.3

Then, (X,τ,E) is a fuzzy soft R_o - space.

Theorem 3.10 In any fuzzy soft R_o - space, every fuzzy soft open set is fuzzy soft closed set.

Proof: Let (F,E) be a fuzzy soft set open set in a fuzzy soft R_0-space. Always, $(F,E)\subseteq\overline{(F,E)}$.....(3)

By hypothesis, $\overline{(F(x,e))} \subseteq (F,E)$, for all fuzzy soft points $F(x.e) \in (F,E)$.

$$\Rightarrow \bigcup \overline{(F(x,e))} \subseteq (F,E)$$

$$\Rightarrow \overline{\bigcup (F(x,e))} \subseteq (F,E)$$

$$\Rightarrow \overline{(F,E)} \subseteq (F,E) \ldots.(4)$$

From (3) and (4), $(F,E) = \overline{(F,E)}$. Therefore, (F,E) is a fuzzy soft closed set.

Theorem 3.11 If (X,τ,E) is a fuzzy soft R_o-space then $\overline{F(x,e)} = fs\ker(F(x,e))$, for all $F(x,e) \in (X,\tau,E)$.

Proof: Let $F(x,e)$ be an arbitrary fuzzy soft point. In a fuzzy soft R_o- space, every fuzzy soft open set is fuzzy soft closed set. Therefore, $\overline{F(x,e)} = fs\ker(F(x,e))$. Since $F(x,e)$ is arbitrary, $\overline{F(x,e)} = fs\ker(F(x,e))$, for all $F(x,e) \in (X,\tau,E)$.

Theorem 3.12 The following statements are equivalent for any two fuzzy soft points $F(x,e)$ and $G(x,e)$ in a fuzzy soft R_o-space.

1. $fs\ker(F(x,e)) \neq fs\ker(G(x,e))$
2. $\overline{F(x,e)} \neq \overline{G(x,e)}$

Proof: It follows from the above result.

4 EXAMPLES ON DEFUZZIFYING

In this section, some results in classical topological space that cannot be extended to fuzzy soft topological space have been exposed by giving suitable examples.

Result 4.1 Let (X,τ) be a topological space and $x \in X$. Then, $y \in \ker(\{x\})$ if and only if $x \in \ker(\{y\})$.

Example 4.2 Let $X = \{x_1, x_2\}, E = \{e_1, e_2\}$, $\tau = \{\phi_E, (F_1,E), (F_2,E), (F_3,E), (F_4,E), 1_E\}$ where $(F_1,E), (F_2,E), (F_3,E), (F_4,E)$ are fuzzy soft sets over X and defined as follows.

(F_1,E)	x_1	x_2
e_1	0.4	0.6
e_2	0.3	0.5

(F_2,E)	x_1	x_2
e_1	0.6	0.4
e_2	0.7	0.5

(F_3,E)	x_1	x_2
e_1	0.6	0.6
e_2	0.7	0.5

(F_4,E)	x_1	x_2
e_1	0.4	0.4
e_2	0.3	0.5

$(F_1,E)^C$	x_1	x_2
e_1	0.6	0.4
e_2	0.7	0.5

$(F_2,E)^C$	x_1	x_2
e_1	0.4	0.6
e_2	0.3	0.5

$(F_3,E)^C$	x_1	x_2
e_1	0.4	0.4
e_2	0.3	0.5

$(F_4,E)^C$	x_1	x_2
e_1	0.6	0.6
e_2	0.7	0.5

Consider the fuzzy soft points

$F_1(x_1,e_1)$	x_1	x_2
e_1	0.4	0
e_2	0	0

$F_1(x_2,e_1)$	x_1	x_2
e_1	0	0.6
e_2	0	0

$$fs\ker(F_1(x_1,e_1)) = (F_4,E)$$

$$fs\ker(F_1(x_2,e_1)) = (F_1,E)$$

Here, $F_1(x_1,e_1) \in fs\ker(F_1(x_2,e_1))$ but $F_1(x_2,e_1) \notin fs\ker(F_1(x_1,e_1))$

Result 4.3 Let (X,τ) be a topological space and $x \in X$. Then, $y \in cl(\{x\})$ iff $x \in cl(\{y\})$.

Example 4.4 Let $X = \{x_1,x_2\}, E = \{e_1,e_2\}$, $\tau = \{\phi_E,(F_1,E),(F_2,E),(F_3,E),(F_4,E),1_E\}$ where $(F_1,E),(F_2,E),(F_3,E),(F_4,E)$ are fuzzy soft sets over X and defined as follows.

(F_1,E)	x_1	x_2
e_1	0.6	0.2
e_2	0.1	0.3

(F_2,E)	x_1	x_2
e_1	0.4	0.8
e_2	0.9	0.7

(F_3,E)	x_1	x_2
e_1	0.6	0.8
e_2	0.9	0.7

(F_4,E)	x_1	x_2
e_1	0.4	0.2
e_2	0.1	0.3

$(F_1,E)^C$	x_1	x_2
e_1	0.4	0.8
e_2	0.9	0.7

$(F_2,E)^C$	x_1	x_2
e_1	0.6	0.2
e_2	0.1	0.3

$(F_3,E)^C$	x_1	x_2
e_1	0.4	0.2
e_2	0.1	0.3

$(F_4,E)^C$	x_1	x_2
e_1	0.6	0.8
e_2	0.9	0.7

Consider the fuzzy soft points

$F_1(x_1,e_1)$	x_1	x_2
e_1	0.6	0
e_2	0	0

$F_1(x_2,e_1)$	x_1	x_2
e_1	0	0.2
e_2	0	0

$\overline{F_1(x_1.e_1)} = (F_2,E)^C$ and $\overline{F_1(x_{2,}e_1)} = (F_3,E)^C$

$F_1(x_2,e_1) \in \overline{F_1(x_1.e_1)}$ but $F_1(x_1,e_1) \notin \overline{F_1(x_{2,}e_1)}$

Result 4.5 A topological space (X,τ) is $\delta - R_o$ space iff for every x and y, $cl(\{x\}) \neq cl(\{y\})$ implies $cl(\{x\}) \cap cl(\{y\}) = \phi$

Example 4.6 From example 4.2

$\overline{F_1(x_1.e_1)} = (F_3,E)^C$ and $\overline{F_1(x_{2,}e_1)} = (F_2,E)^C$

Therefore, $\overline{F_1(x_1.e_1)} \neq \overline{F_1(x_{2,}e_1)}$. But $\overline{F_1(x_1.e_1)} \cap \overline{F_1(x_{2,}e_1)} \neq \phi_E$

Result 4.7 A topological space (X,τ) is a $\delta - R_o$ space if and only if for every x and y, $\ker(\{x\}) \neq \ker(\{y\})$ $\ker(\{x\}) \cap \ker(\{y\}) = \phi$.

Example 4.8 Consider the example 4.4

$fs\ker(F_1(x_1,e_1)) = (F_1,E)$
$fs\ker(F_4(x_2,e_1)) = (F_4,E)$

$fs\ker(F_1(x_1,e_1)) \neq fs\ker(F_4(x_2,e_1))$ and
$fs\ker(F_1(x_1,e_1)) \cap fs\ker(F_4(x_2,e_1)) \neq \phi_E$

CONCLUSION

The concept of kernels in fuzzy soft topological space and fuzzy soft R_0-space were introduced. Some theorems on fuzzy soft kernels with some examples were discussed and this study could be continued to give more result in future.

REFERENCES

Chang C.L (1968), Fuzzy topological spaces, *Journal of Mathematical Analysis and Applications*, **vol. 24**, no. 1, pp. 182-190

Mahanta J and Das P.K,(2012), Results on Fuzzy Soft Topological Spaces, arXiv:1203.0634v1.

Maji P.K, Biswas R and Roy A.R, (2001), *Journal of Fuzzy Mathematics*, **vol. 9**, no. 3, pp. 589-602.

Molodstov D(1995), Soft Set Theory - First Results, Computers Math. Appl. 37(4/5) 19-31.

Palaniappan N (2002), Fuzzy Topology, Narosa Publications,.

Pazar Varol B and Aygun H (2012), Fuzzy soft topology, *Hacettepe Journal of Mathematics and Statistics*, **vol. 41**, no. 3, pp. 407-419.

Shabir M and Naz M (2011), On soft topological spaces, *Computers and Mathematics with Applications*, **vol. 61**, no. 7, pp.1786-1799.

Tanay B and Kandemir M.B(2011), *Topological structure of fuzzy soft sets, Computers and Mathematics with Applications*, **vol. 61**, no. 10, pp. 2952-2957.

Zadeh L.A 8(1965), Fuzzy Sets, Information and Control, 338-353.