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GENERALIZED CLOSED SETS IN IDEAL TOPOLOGICAL SPACES

MUTHUMARI.S[1] AND ANBUCHELVI.M[2]

[1]M.Phil. Scholar
[2]Associate Professor
Department of Mathematics
V.V.Vanniaperumal College for Women
Virudhunagar. TamilNadu, India.

ABSTRACT

The aim of this paper is to introduce a new class of generalized closed sets in ideal topological space via a- open sets.

Key-words: a- open set, ∗- closed set, I_a- closed sets.

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INTRODUCTION

Levin [Levine, 1970], introduced the notion of generalized closed sets in topological space. The concept of ideal topological space was introduced by Kuratowski [Kuratowski, 1966] and Vaidyanathaswamy[Vaidyanathaswamy]. In 1999, the notion of I_g – closed set was introduced by Dontchev [Dontchev et al., 1999]. Further investigation and characterization of I_g- closed sets had been developed by Navaneetha Krishnan and Joseph [Navaneetha Krishnan and Paulraj Joseph, 2008] Yukser, Acikgoz and Noiri [Yukser and Noiri, 2005] studied δ-I closed sets. In (1999) Ekici [Erdal Ekici, 1999] introduced the notion of a- open sets in topological space. The purpose of this paper is to define I_a- closed sets and study some basic properties.

Preliminaries:

Definition 2.1 [Kuratowski, 1966]An ideal I on X is a collection of subsets of X satisfying the following

1. If $A \in I$ and $B \subseteq A$ then $B \in I$
2. If $A \in I$ and $B \in I$ then $A \bigcup B \in I$

Definition 2.2 [Jankovic and Hamlet, 1990] A subset of a topological space (X,τ,I) is said to

be $*$- closedif $B \subseteq A$

Definition 2.3 A subset of a topological space (X, τ) is said to be

1. α-open [Njastad, 1965] if $A \subseteq$ int (cl (int (A))).
2. semi- open [Levine, 1963] if $A \subseteq$ cl (int (A))).
3.regular open [Stone,1937] if A = int(cl(A)).
4. a-open [Erdal Ekici, 1999S] if $A \subseteq$ int (cl (int_δ (A))).
5. g - closed [Ravi et al., 2011] if cl (A) $\subseteq U$ whenever $A \subseteq U$ and U is open in X.
6. gδ – closed [Muthulakshmi et al.] if cl (A)$\subseteq U$ whenever $A \subseteq U$ and U is δ – open in X.
7. αg – closed [Maki et al., 1994] if cl_α (A) $\subseteq U$ whenever $A \subseteq U$ and U is open in X.

The complement of a α-open (resp.semi-open, regular-open, a-open) set is called α-closed (resp.semi-closed, regular-closed, a-closed).

Definition 2.4[Velico, 1968]A subset A of a space (X,τ) is called a δ- closed set if $A=cl_\delta(A)$ whenever $cl_\delta(A) = \{x \in X$: int(cl(U)) $\bigcap A \neq \Phi$, $U \in \tau$ and U is open in X$\}$. The complement of a δ- closed set isδ-open in X.

Definition 2.5 A subset A of a space (X,τ,I) is said to be

1. αIg – closed [Maragatavalli and Vinothini, 2014] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is α- open in X.
2. $I\hat{g}$ – closed [Antony Rex Rodrig et al., 2011]if $A^* \subseteq U$ whenever $A \subseteq U$ and U is semi – open in X
3. I_g – closed [Navaneetha Krishnan and Paulraj Joseph, 2008] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in X.
4. 4.$I_{g\delta}$ – closed [Ravi et al., 2011] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is δ-open in X.
5. I_{rg} – closed[Navaneetha Krishnan and Sivaknraj, 2010] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
6. $I_{\alpha gg}$- closed if[Ravi et al., 2011] $A^* \subseteq U$ whenever $A \subseteq U$ and U is αg-open in X.

Results 3.22 In a topological space (X,τ),

1. Every a- open set is semi-open [Jankovic and Hamlet, 1990]

2. Every a-open set is α- open [Esref Hatir Seluck, 2009]
3. Every regular–open set is a-open [Esref Hatir Seluck, 2009]
4. Every δ- open set is a- open [Erdal Ekici, 1999]

3 I_a-GENERALIZED CLOSED SETS

Definition 3.1 A subset A of an ideal topological space (X, τ,I) is said to be I_a-closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is a- open in X. The complement of an I_a-closed set is an I_a- open in X.

Example 3.2 Let X = {a,b,c, d}, τ = {φ, {a}, {b}, {a, b}, X } and I ={φ}. The collection {φ, {c,d}, {a,c,d}, {b,c,d}, X} is the set of all I_a- closed sets.

Theorem 3.3 Every element of I is I_a – closed set in an ideal topological space X.

Proof: Let $A \in I$ be arbitrary. For any $U \in \tau$, $U \cap A \subseteq A$. By the definition of an ideal, $U \cap A \in I$. Therefore, A* =φ. If $A \subseteq U$ for any a- open set U, then $A^* = \varphi \subseteq U$. So, A is I_a-closed set in X.

Theorem 3.4 Every ∗-closed set is I_a-closed set in an ideal topological space (X, τ,I).

Proof Let A be any ∗-closed set in X. Then, $A^* \subseteq A$. Therefore, for any a-open set U of X, $A \subseteq U$ implies that $A^* \subseteq U$. Therefore, A is I_a- closed set in X.

Remark 3.5 The following example shows that there is I_a- closed set that is not a ∗-closed set in an ideal topological space (X, τ, I) .

Example 3.6 Let X = {a, b, c}, τ = {φ,{a}, X} and I = {φ,{b}}. Then,{c} is I_a- closed but not a ∗-closed set.

Theorem 3.7 For every subset A of an ideal topological space X, A* is always a I_a-closed set in X.

Proof: Let $A^* \subseteq U$ where U is any a- open set in X. By [14] $(A^*)^* \subseteq A^*$. Then $(A^*)^* \subseteq U$. Hence, A* is I_a- closed.

Theorem 3.8 Every a- open set that is I_a-closed is always ∗ - closed set in an ideal topological space X.

Proof: Assume that A is both I_a – closed and a- open set in X. Clearly $A \subseteq A$ and A is a- open in X. Since A is I_a- closed set, $A^* \subseteq A$. Hence A is ∗- closed.

Theorem 3.9 For every x is an ideal topological space in X, either {x} is a-closed or $\{x\}^c$is I_a – closed.

Proof. Suppose {x} is not a-closed, then $\{x\}^c$is not a-open. Now, the only a- open set containing $\{x\}^c$ is X. Therefore $(\{x\}^c)^* \subseteq X$, and hence $\{x\}^c$ is I_a-closed set in X.

Theorem 3.10 Every $I_g^{\wedge}$-closed set is I_a-closed in an ideal topological space X.

Proof Let A$\subseteq$U where U is a-open in X. By[17], every a-open set is semi open in X.Now A$\subseteq$U where U is semi-open in X.By hypothesis,A*$\subseteq$U and hence A is I_a-closed in x.

Remark 3.11 The converse is not from the following example.

Example 3.12 Let X={a,b,c} τ={φ,{a},{b},{a,b},{a,c},X} and I={φ,{b}}.Then {a,b} is a I_a-closed set but not a $I\hat{g}$ - closed set.

Theorem 3.13 Every αIg-closed set is I_a-closed in an ideal topological space X.

Proof: Suppose thatA is any αIg-closed set in X.Let A$\subseteq$U where U is a –open in X.By [21] every a-open set is α-open.Now A$\subseteq$U where U is α-open in X.By hypothesis,A*$\subseteq$ U and hence A is I_a-closed set in X.

Remark 3.14 The followingexample establishes that the converse is not true.

Example 3.14 Let X={a,b,c}, τ ={φ,{a},{b},{a,b},{a,c},X} and I={φ}. Then, {a,b} is I_a- closed but not αIg-closed set.

Theorem 3.16 Every I_a-closed set is I_{rg}-closed in an ideal topological space X.

Proof : Assume that A is any I_a-closed set in X.Let U be any regular open set such that A $\subseteq$U. By [17] , every regular open set is a-open set in X.Now A$\subseteq$U where U is a-open in X.By hypothesis, A*$\subseteq$U,and hence A is I_{rg}-closed set in X.

Example 3.17 Let X={a,b,c},τ={φ,{a},{b},{a,b},X} and I={φ} Then {a,b} is I_{rg}-closed set but not I_a-closed.

Theorem 3.18 Every $I_{\alpha gg}$-closed set is I_a-closed in an ideal topological space X.

Proof: Assume that A is any $I_{\alpha gg}$-closed set in X.Let A $\subseteq$U where U is a-open set in X.[10] every a-open set is αg-open in X.By hypothesis,A*$\subseteq$U and hence A is I_a-closed set in X.

Example 3.19 This example shows that there is an I_a-closed set which is not a $I_{\alpha gg}$-closed set in X.Let X={a,b,c,d} τ={φ,{a},{b},{a,b},{a,b,c},{a,b,d},X} and I={φ}.Then {a,c} is I_a-closed but not $I_{\alpha gg}$-closed.

Theorem 3.20 Every I_a- closed set is $I_{g\delta}$-closed in an ideal topological space in X.

Proof: Let A$\subseteq$U where U is δ-open set in X. By [3] every δ-open set is a-open set in X. By hypothesis, A*$\subseteq$U and hence A is $I_{g\delta}$ -closed.

Example3.21 Let X= {a,b,c,d}, τ ={φ,{a},{b},{a,b},{a,b,c},{a,b,d},X} and I= {φ}. Then {a,d} is a $I_{g\delta}$- closed but not a I_a-closed set.

CONCLUSION

The concept of generalized closed sets in an ideal topological space has been defined with the help of a-open sets. Relations among existing closed setin ideal topological space and I_a- closed set have been derived.

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