International Multidisciplinary Innovative Research Journal
-An International refereed e-journal

ISSN: 2456 – 4613
Volume – IV Issue - 1
November - 2019

α_1, α_2- GAMMA NEAR RINGS

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ABSTRACT

In this paper, we have defined α_1, α_2- gamma near rings. It is proved that M/I is aα_1-gamma near ring for every ideal I of a α_1-gamma near ring M. Also, we showed that every α_1-gamma near ring M is isomorphic to a sub direct product of sub directly irreducible α_1-gamma near ring and α_1-gamma near ring has Pierce Decomposition with constant gamma near ring and zero symmetric gamma near ring. We showed that every regular gamma near ring is α_2-gamma near ring. As a characterization it is proved that a gamma near ring M is α_2-gamma near ring if and only if every M -gamma subgroup contains a non-zero idempotent.

MATHEMATICS SUBJECT CLASSIFICATION: 16Y30

Keywords: Gamma near ring, idempotent, nilpotent, sub directly irreducible, weak commutative, zero symmetric.

INTRODUCTION

For basic notations and definitions of near rings we refer Pilz(1983). A generalization of both the concepts near rings and gamma rings, namely gamma near rings was introduced by Satyanarayana Bhavanari (1999). Yong Uk Cho (2006) obtained some basic concepts and properties of gamma near ring through regularity condition. Analogous to the concept of regular gamma near ring we define α_1-gamma near ring and α_2-gamma near ring.

PRELIMINARIES

DEFINITION:2.1[4]

A right near ring is a set N together with two binary operators '+' and '.' such that

i) (N,+) is a group (not necessarily abelian)

ii) (N,.) is a semigroup

iii) For all x,y,z in N, $(x+y)z = xz + yz$ (right distributive law)

DEFINITION:2.2[23]

Let $(M,+)$ be a group (need not be abelian) and Γ be a non-empty set. Then M is said to be $\boldsymbol{\Gamma - nearring}$if there exists a mapping $M \times \Gamma \times M \to M$ (the image of (a,α,b) is denoted by $a\alpha b$) satisfying the following condition :

i) $(a+b)\alpha c = a\alpha c + b\alpha c$

ii) $(a\alpha b)\beta c = a\alpha(b\beta c) \forall a,b,c \in M and \alpha,\beta \in \Gamma$.

DEFINITION:2.3[13]

A Γ-near ring M is said to be ***zero symmetric Γ-near ring***if,$a\gamma 0 = 0$ for all$a \in M \& \gamma \in \Gamma$.

NOTATION:2.4[13]

$M_0 - \{a \in M | a\gamma 0 = 0\}$ which is called the ***zero symmetric part*** of M.

$M_c - \{a \in M | a\gamma 0 = a\}$ which is called the ***constant part*** of M.

DEFINITION:2.5 A gamma near ring M is called ***regular*** $\boldsymbol{\Gamma - nearring}$if for any element $a \in M$, there exists an element x in M, such that $a = a\gamma_1 x\gamma_2 a$ for every non-zero pair of elements $\gamma_1, \gamma_2 \in \Gamma$. And the element a is called ***regular***.

DEFFINITION:2.6

An element e in the gamma near ring, M is called ***idempotent***,if $e\gamma e = e$ for all γ in Γ.

DEFINITON:2.7

A near ring N is called ***weak commutative***if,$xyz = xzy$ for every x,y,z in N.

DEFINITION:2.8

The gamma near ring M is ***sub directly irreducible***, if and only if the intersection of the non-zero ideals of M is non-zero.

DEFINITION:2.9

Let (M,+) be a sub group (need not be abelian) and Γ be a non-empty set. Then M is said to be$\boldsymbol{sub\Gamma - nearring}$ if there exists a mapping $M \times \Gamma \times M \to M$ (the image of (a,α,b) is denoted by $a\alpha b$) satisfying the following condition:

i) $(a+b)\alpha c = a\alpha c + b\alpha c$

ii) $(a\alpha b)\beta c = a\alpha(b\beta c) \forall a,b,c \in M and \alpha,\beta \in \Gamma$.

α_1, α_2- Gamma Near rings

DEFINITION:3.1

Let M be a right gamma near ring. If i) for every a in Mthere exists x in M such that $a = x\gamma a\gamma x$ for all $\gamma \in \Gamma$ then we say that M is a$\boldsymbol{\alpha_1}$**-gamma near ring**.

ii) for every a in M^* (i.e.)$M - \{0\}$ there exists x in $M - \{0\}$ such that $x = x\gamma a\gamma x$ for all $\gamma \in \Gamma$ then we say that Mis a α_2-gamma near ring.

PROPOSITION:3.2

In a α_1 gamma near ring for every ain M there exists some x in M such that i)$\alpha^2\gamma x = x\gamma\alpha^2$ ii)$a = x^n\gamma a\gamma x^n$ for all $\gamma \in \Gamma$.

Proof:

i) Let M be a α_1-gamma near ring and $a \in M$,x in M such that $a = x\gamma a\gamma x$ for all$\gamma \in \Gamma$. This implies $x\gamma a^2 = (x\gamma a)\gamma a = x\gamma a\gamma(x\gamma a\gamma x) = (x\gamma a\gamma x)\gamma a\gamma x = a\gamma a\gamma x = a^2\gamma x$.Hence $x\gamma a^2 = a^2\gamma x$ for every a in M and for all $\gamma \in \Gamma$ and the result follows.

ii) $x\gamma a\gamma x = x\gamma(x\gamma a\gamma x)\gamma x = x^2\gamma a\gamma x^2 = \cdots\cdots\cdots = x^n\gamma a\gamma x^n$for all $n \geq 1$ and $\gamma \in \Gamma$. Hence the result.

PROPOSITION:3.3 *Let M be a regular gamma near ring. If M is weak commutative then M is a α_1- gamma near ring.*

Proof: Since M is a regular gamma near ring for every $a \in M$ there exists $b \in M$such that $a\gamma b\gamma a = a$ and $\gamma \in \Gamma$. Let $x = a\gamma b$ then $x\gamma a\gamma x = (a\gamma b)\gamma a\gamma(a\gamma b) = (a\gamma b\gamma a)\gamma a\gamma b = a(\gamma a\gamma b) = a\gamma b\gamma a = a$ for every a in M and$\gamma \in \Gamma$ and M becomes α_1-gamma near ring. Hence proved.

PROPOSITION:3.4

Let M be a zero symmetric α_1-gamma near ring. If M is weak commutative then for any a, b in M,$\gamma \in \Gamma$,$a\gamma b = 0$ implies $b\gamma a = 0$.

Proof:

Suppose $a\gamma b = 0$ for $a, b \in M$ and $\gamma \in \Gamma$. Since M is a α_1-gamma near ring there exists $x, y \in M$ such that $x\gamma a\gamma x = a$ and $y\gamma b\gamma y = b$ for all $\gamma \in \Gamma$. Also, since M is weak commutative $b\gamma a = (y\gamma b\gamma y)\gamma(x\gamma a\gamma x) = y\gamma b\gamma(y\gamma x\gamma a)\gamma x = y\gamma b\gamma(y\gamma a\gamma x)\gamma x = y\gamma(b\gamma y\gamma a)\gamma x^2 = y\gamma(b\gamma a\gamma y)\gamma x^2 = (y\gamma b\gamma a)\gamma y\gamma x^2 = y\gamma(a\gamma b)\gamma y\gamma x^2 = y\gamma(0)\gamma y\gamma x^2 = 0$.Hence the result.

PROPOSITION:3.5

Homomorphic image of aα_1-gamma near ring is also a α_1-gamma near ring.

Proof:

The result is obvious.

PROPOSITION:3.6 If *I is an ideal of the α_1-gamma near ring M then M/I is also an α_1-gamma near ring.*

Proof:

The function $\varphi: M \to M/I$ be defined by $\phi(x) = I + x$ is an epimorphism. Since M is a α_1-gamma near ring, M/I is also a gamma near ring. Let $a + I, b + I \in M/I$, $a + I = b + I \Rightarrow a - b \in I$ we have $\phi(a - b) = 0$. Thus $\phi(a) - \phi(b) = 0$. Hence $\phi(a) = \phi(b)$. Thus ϕ is well-defined. Hence the result.

PROPOSITION: 3.7

Every α_1-gamma near ring M is isomorphic to a sub direct product of sub directly irreducible α_1-gamma near ring.

Proof:

M is isomorphic to a sub direct product of sub directly irreducible gamma near rings M_i's and each M_i is a homomorphic image of M under the projection map π_i. By Proposition:3.5 M_i's are gamma near ring.

THEOREM:3.8

M is a α_1-gamma near ring if and only if every x in M can be written as $a = u + v$ where $u \in M_0$ and $v \in M_c$ and $u = x_0\gamma(n\gamma x + m) - x_0\gamma a\gamma x_c$, $v = x_0\gamma a\gamma x_c + x_c$, $x = x_0 + x_c \in M_0 \oplus M_c$ where $x_0, n \in M_0$ and $x_c, m \in M_c$. Further more $u \in M_0$ and $v \in M_c$.

Proof:

For the 'only if ' part, let $a \in M$. Since M is α_1-gamma near ring there exists x in M such that $a = x\gamma a\gamma x\gamma \in \Gamma$ also by using Peirce Decomposition, we can write $a = n + m$ and $x = x_0 + x_c$ where $x \in M, n, x_0 \in M_0$ and $m, x_c \in M_c$. Now, $a = (x_0 + x_c)\gamma(n + m)\gamma x = (x_0 + x_c)\gamma(n\gamma m + m\gamma x) = (x_0 + x_c)\gamma(n\gamma x + m) = x_0\gamma(n\gamma x + m) + x_c\gamma(n\gamma x + m) = x_0\gamma(n\gamma x + m) + x_c = x_0\gamma(n\gamma x + m) - x_0\gamma a\gamma x_c + x_0\gamma a\gamma x_c + x_c$. Thus $a = u + v$ where $u = x_0\gamma(n\gamma x + m) - x_0\gamma a\gamma x_c$ and $v = x_0\gamma a\gamma x_c + x_c$. Now, $u\gamma 0 = [x_0\gamma(n\gamma x + m) - x_0\gamma a\gamma x_c]\gamma 0 = x_0\gamma(n\gamma x + m)\gamma 0 - x_0\gamma a\gamma x_c\gamma 0 = x_0\gamma(n\gamma x\gamma 0 + m\gamma 0) - x_0\gamma a\gamma x_c\gamma 0 = x_0\gamma(n\gamma x\gamma 0 + m) - x_0\gamma a\gamma x_c = x_0\gamma(n\gamma x_c + m\gamma x_c) - x_0\gamma a\gamma x_c$. Since $m \in M_c$ and $x\gamma 0 = x_c$, we have $u\gamma 0 = 0$. Therefore, $u \in M_0$ and $v\gamma 0 = (x_0\gamma a\gamma x_c + x_c)\gamma 0 = x_0\gamma a\gamma x_c\gamma 0 + x_c\gamma 0 = x_0\gamma a\gamma x_c + x_c = v$. This gives $v\gamma 0 = v$. Therefore $v \in M_c$. Thus $a = u +$

v, where $u \in M_0$ and $v \in M_c$. This completes the proof of the 'only if' part.

For the 'if part' we assume for every a in M with $a = u + v$ where $u \in M_0, v \in M_c$ with $u = x_0\gamma(n\gamma x + m) - x_0\gamma a\gamma x_c$ and $v = x_0\gamma a\gamma x_c + x_c$ where $x = x_0 + x_c, x_0, n \in M_0$ and $x_c, m \in M_c$. Now we shall show that M is a α_1-gamma near ring , we have $a = u + v = x_0\gamma(n\gamma x + m) - x_0\gamma a\gamma x_c + x_0\gamma a\gamma x_c + x_c = x_0\gamma(n\gamma x + m\gamma x) + x_c = x_0\gamma(n + m)\gamma x + x_c = x_0\gamma a\gamma x + x_c\gamma a\gamma x_c = (x_0 + x_0)\gamma a\gamma x = x\gamma a\gamma x$. Thus for every a in M, $a = x\gamma a\gamma x$ some x in M. Hence the theorem.

PROPOSITION:3.9

In a α_2-gamma near ring, $E \neq \{0\}$.

Proof:

Let M be a α_2-gamma near ring. So for every a in M^* such that $x\gamma a\gamma x = x$ for all $\gamma \in \Gamma$. Here $(a\gamma x)^2 = a\gamma x\gamma(a\gamma x) = a\gamma(x\gamma a\gamma x) = a\gamma x$ and $(x\gamma a)^2 = x\gamma a\gamma(x\gamma a) = (x\gamma a\gamma x)\gamma a = x\gamma a$. (i.e) $a\gamma x$ and $x\gamma a$ are idempotents for some $\gamma \in \Gamma$ and hence $E \neq \{0\}$. Hence the result.

PROPOSITION:3.10

Every regular gamma near ring is a α_2-gamma near ring.

Proof:

Let M be a regular gamma near ring. Hence for every a in M there exists b in M such that $a\gamma b\gamma a = a$ for all $\gamma \in \Gamma$. Let $x = b\gamma_1 a\gamma_2 b$ for some $\gamma_1, \gamma_2 \in \Gamma$. Now $x\gamma a\gamma x = (b\gamma_1 a\gamma_2 b)\gamma a\gamma(b\gamma_1 a\gamma_2 b) = b\gamma_1(a\gamma_2 b\gamma a)\gamma b\gamma_1 a\gamma_2 b = b\gamma_1(a)\gamma b\gamma_1 a\gamma_2 b = b\gamma_1(a\gamma b\gamma a)\gamma b = b\gamma_1 a\gamma_2 b = x$. This proves that every regular gamma near ring is a α_2-gamma near ring. Hence the theorem.

DEFINITION:3.11

A sub gamma near ring M' of a gamma near ring M is called a **α_2- sub gamma near ring** if for every a in $M^* = M - \{0\}$ there exists an x in M^* such that $x\gamma a\gamma x = x$ for all $\gamma \in \Gamma$.

PROPOSITION:3.12

Let M be a α_2-gamma near ring then,

i) Every invariant subgroup M' of M is also an α_2-sub gamma near ring.
ii) Every ideal I of a zero symmetric α_2-gamma near ring M is also an α_2-sub gamma near ring.

Proof:

i) Let $a \in M^*$. Since M is an α_2-gamma near ring there exists x in M such that $x = x\gamma a\gamma x$ for all $\gamma \in \Gamma$. Now M' is an invariant subgroup of M implies $x\gamma a\gamma x \in M'$ (i.e.) $x \in M'$. Consequently M' is a α_2-sub gamma near ring.

ii) Let I be an ideal of the zero symmetric α_2-gamma near ring N. Let $a \in I^* = I - \{0\}$. Since N is anα_2-gamma near ring there exists $x \in N^*$such that $x\gamma a\gamma x = x$ for all $\in \Gamma$. Now $x\gamma a\gamma x \in N^*\Gamma I^*\Gamma N^* \subseteq I^*$. Thus the desired result follows.

THEOREM:3.13

A gamma near ring M is α_2 if and only if every M gamma subgroup contains a nonzero idempotent.

Proof:

Suppose M be a α_2-gamma near ring. If M' is a M gamma subgroup and $a \in M$ then there exists $x \in N$ such that $x\gamma a\gamma x = x \neq 0$.As in the proof of Proposition:3.9 we see that $x\gamma a \in M$ is an idempotent and $x\gamma a \neq 0$. For the 'if part' let $a \in M$. Clearly $M\Gamma a$ is a subgroup of M. Now, by hypothesis $M\Gamma a$ contains a nonzero idempotent e. Then $e = b\gamma a$ for some b in M.Suppose $x = b\gamma a\gamma b$.Clearly$x \neq 0$. We have $x\gamma a\gamma x = b\gamma a\gamma b\gamma a\gamma(b\gamma a\gamma b) = b\gamma a\gamma b\gamma a\gamma b\gamma a\gamma b = e\gamma e\gamma e b = e\gamma b = (b\gamma a)\gamma b = x$ and this completes the proof.

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