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A THEORETICAL STUDY OF RIGHT BIPOTENT NEAR RINGS

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ABSTRACT

A right bipotent near ring is an important algebraic structure in the study of Abstract Algebra and near-ring theory. The present study aims to examine the structural characterization and algebraic behaviour of right bipotent near rings under various operational conditions such as intra regularity, reversibility etc. Particular emphasis is placed on the interaction between bipotency and other significant near-ring properties, including regularity, right S-unital, quasi weak commutativity and zero symmetry In addition, several behavioural properties of right bipotent near rings is also discussed leading to the major contribution in the theory of near rings.

Keywords: Intra regular, Quasi weak commutativity, Right bipotentcy, SR near ring, Strongly regular.

INTRODUCTION

A near ring N is defined to be right bipotent if $aN = a\ 2N$ for each a in N. In this paper, we have proved some more results on right bipotent near rings by using the concepts of S' - near ring; Boolean condition regularity and so on. Epimorphic image of a right bipotent near ring is obtained. Furthermore, it is proved that every right bipotent near ring with P_3 property is right S-unital iff it possesses a mate function in N. And every right bipotent near ring is right regular under regularity and Boolean under strong regularity. The reversible property of right bipotent near rings is obtained under quasi weak commutativity.

PRELIMINARIES

Definition 2.1 [7]

A **right near-ring** (N, +, .) is a non-empty set together with two binary operations '+' (called addition) and '·' (called multiplication) such that:

(i) (N, +) is a group (not necessarily abelian)

(ii) (N, .) is a semi group

(iii) $(x+y).z = x.z + y.z, \forall x, y, z \in N$.

Definition 2.2 [6]

A function 'f' from N to N is called a **mate function** for N if x = xf(x)x, for all 'x' in N and f is called as mate of x.

Definition 2.3 [3]

A mapping f: N ⟶ N' is said to be a **homomorphism**, where N and N' are near rings, when the following conditions are satisfied:

i. f(m+n) = f(m) + f(n)

ii. f(mn) = f(m)f(n) where m, n belongs to the near ring N.

Definition 2.4 [3]

A (right) near ring N is called **subdirectly irreducible** if N is not isomorphic to a non-trivial subdirect product of near rings

Definition 2.5[3]

A near ring N is called **boolean** if $a = a^2$, for all a in N.

Definition 2.6 [13]

A near ring N is said to be **quasi weak commutative** if xyz = yxz, for all x,y,z in N.

Definition 2.7 [1]

A right near ring N is said to be **regular** if aba = a for all a, b in N.

Definition 2.8 [7]

N is said to hold **P_3 property** if abc = acb holds good for all a, b,c in N.

Definition 2.9[13]

A right near ring N is said to be **α_1 near ring** if for all a in N, there exists x in N, such that xax = a.

Definition 2.10 [12]

A right near ring is called **left strongly regular (right strongly regular)** if for every a in N there is an x in N such that $a = xa^2$ ($a = a^2x$).

Definition 2.11 [5]

A right near ring is called **left regular (right regular)** if for every a in N there is an x in N such that $a = xa^2$ ($a = a^2x$) and a = axa. A near ring is said to be **strongly regular**, if it is both left and right strongly regular.

Definition 2.12 [2]

A near ring is said to be **intra regular** if for every $a \in N$, there exists $x,y \in N$ such that $a = xa^2y$.

Proposition 2.13 [2]

Every regular near ring is intra-regular near ring.

Definition 2.14 [1]

A **reversible near-ring N** is an algebraic structure where the product ab = 0 implies ba = 0, for all a,b in N

Definition 2.15 [13]

A near ring N is called as **SR near ring** if for every a in N, there exists x in N*, such that

axax = ax

Definition 2.16 [7]

A right near ring N is said to be **right bipotent** if $a^2N = aN$, ∀ a ∈ N.

Example 2.17

The near ring $(N, +, .)$ where $(N, +)$ is the group of integers modulo 4 and multiplication by the following table:

'.'	**0**	**1**	**2**	**3**
0	0	0	0	0
1	0	3	0	1
2	0	2	0	2
3	0	1	0	3

is a right bipotent near ring.

MAIN RESULTS

Theorem 3.1

Epimorphic image of a right bipotent near ring 'N' is also so.

Proof:

Given that, N is a right bipotent near ring. Then $aN = a^2N$, for all 'a' in N ---(1)

Let m: N ⟶ N' be an epimorphism where N and N' are right bipotent near rings.

Let b' ∈ N'. Then, there exists b ∈ N such that m(b) = b'.

Also, for n' ∈ N', ∃ n ∈ N such that f(n) = n'

Now, b'n' = m(b)m(n)

=m(bn)(since, m is homomorphic)

$= m(b^2n_1)$ (by (1)

$= m(b^2)\, m(n_1)$

$= (m(b))^2 m(n_1)$

$= (b')^2 n_1'$

That is, $b'n' = b'^2 n_1'$

$\Longrightarrow b'n' \in b'^2 N_1'$

Hence, $b'N' \subseteq b'^2N'$ and similarly, $b'^2N' \subseteq b'N'$. Thus, $b'N' = b'^2N'$. Therefore, N' is also a right bipotent near ring, which completes the proof.

Corollary 3.2

Let N be a right bipotent near ring. If I is any ideal of N, then N/I is also a right bipotent near ring.

Proof:

Let g: N ⟶ N' be canonical near ring homomorphism defined by g(x) = I + x, where N and N' are right bipotent near rings.

Clearly, g is onto. Then, proof follows by theorem 3.1

Theorem 3.3

Every right bipotent near ring with P_3 property is right S-unital iff it possesses a mate function in N.

Proof:

Let N be a right bipotent near ring.

Given: N is right S-unital (i.e:) $x \in xN$.

$\Rightarrow x = xn$, for some n in N

$= xf(x)$ (Put $n = f(x)$)

$= x^2f(x)$ (since, N is right S-unital)

$= xxf(x)$

$= xf(x)x$

$\Rightarrow x = xf(x)x$, for all $x \in N$, where f is a mate function in N.

Therefore, N has a mate function in N.

Conversely, given, N has a mate function.

Then, $x = xf(x)x$

$= xxf(x)$ (by P_3 property)

$= x^2f(x)$

$= xf(x)$, for some $f(x) \in N$

$\Rightarrow x \in xN$

Therefore, N is right S-unital near ring.

Theorem 3.4

Every right bipotent near ring N is isomorphic to subdirect product of sub-directly irreducible right bipotent near rings.

Proof:

Let N be a right bipotent near ring. By theorem 2.17 of Pilz, N is isomorphic to sub-direct product of sub-directly irreducible near rings N_i's. And each N_i is a homomorphic image of N under the projection map π_i. Now, proof follows from Theorem 3.1.

Theorem 3.5

Let N be a right bipotent α_1 near ring. Then every quasi weak commutative boolean near ring with regularity is left singular and right strongly regular.

Proof:

Let N be a right bipotent near ring. Then, $x \in aN = a^2N$.

Then $x = a^2y$, for some $y \in N$.

Consider $x = a^2y$

$= ay$ (since, N is Boolean)

$= (axa)y$ (since, N is regular)

$= ax(ay)$

$= ax(x) = ax^2 = xax$ (since, N is quasi weak commutative)

$= a$ (when $x = 1$)

$\Rightarrow ay = a$ and $ay = a^2y$

$\Rightarrow a^2y = a$

Therefore, N is left singular and right strongly regular.

Theorem 3.6

Every SR near ring with quasi weak commutativity in a right bipotent near ring is regular if cancellation law holds good.

Proof:

Let N be a right bipotent near ring. Let $x \in aN = a^2N$.

Then $x = ay = a^2y$ --- (1), for some $y \in N$

Consider, $yaya = y(ay)a$

$= yxa$ (by (1))

$= y(xaxa)$ (N is SR near ring)

= yxaxa

= y(x)axa

= y(a^2y) axa (by (1))

= ya(ay)axa

= ya(x)axa (given)

= y(axax)a

= y(ax)a (N is SR near ring)

= (yax)a

=(xya)a (N is quasi weak commutative near ring)

= x(yaa)

= x(aya) (N is quasi weak commutative near ring)

= x(ay)a

= xxa (given)

= x^2a

⇒ yaya = x^2a

⇒ yay = x^2 (by R.C.L)

⇒ yaya = $(a^2y)^2$

⇒ ya= a^4y (by R.C.L)

⇒ ya = $a^2(a^2y)$

= a^2x (given)

⇒ ya = aax

= xaa (N is quasi weak commutative)

⇒ y = xa (by R.C.L)

⇒ y = (ay)a (given)

⇒ y = aya, $\forall a, y \in N$

Hence N is regular near ring which is to be proved.

Theorem 3.7

Every right bipotent near ring is right regular under regularity.

Proof:

Let N be a right bipotent near ring. Then xN = x^2N, $\forall x \in N$

Let x ϵ a^2N. Then x = a^2y, for some y ϵ N

Consider, x = a^2y

= a(ay)

= aya(ay) (since, N is regular)

= ay(a)ay

= ay(xa^2y)ay(by prop. 2.13)

= ayxa(aya)y

= ayxaay (since, N is regular)

= ay(xa^2y)

= aya (by proposition 2.13)

= a

⇒ x = a^2y = a

Hence, for all a, y in N, a^2y = a and a = axa (given) which completes the proof that N is right regular near ring.

Proposition 3.8

Every right bipotent near ring is boolean under strong regularity

Proof:

Let N be a right bipotent near ring, (i.e;) xN = x^2N, for all x in N

Then x = ay = a^2y, for some y in N

Consider, x = ay

= (xa^2)y (N is strongly regular)

= xa^2y

$= xx$

$= x^2$

$\Rightarrow x = x^2$, for all x in N.

Theorem 3.9

Every right bipotent near ring is reversible under quasi weak commutativity

Proof:

Let N be a right bipotent near ring, (i.e;) $xN = x^2N$, for all x in N

Let $b \in aN$. Then $b = ax$, for some $x \in N$

Given that ab = 0, for all a,b in N. Now it is enough to prove that ba = 0, for all a,b in N.

Consider, ba = (ax)a

= axa

=aax (N is quasi weak commutative)

= a(ax)

= ab (since, ab = x)

= {0} (given)

Hence, N is reversible near ring.

CONCLUSION

An analytical study over right bipotent near rings led to many more extensive results when interacted with regular structures, especially right regularity, right strongly regular and left singularities. Subdirect product of irreducible right bipotent near rings is also proved which led to the further extension of any other epimorphic structures between near rings. Quasi weak commutativity plays a major role in right bipotentcy which made wide openings in cyclic weak commuativity and quasi cyclic weak commuative substructures interacted with right bipotent near rings. Thus, role of right bipotentcy plays a vital role in diverse fields of algebraic substructures and provides a structural framework for other near ring theories.

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