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ANALYSIS OF OUTSTANDING FACULTY IN SINGLE VALUED NEUTROSOPHIC ENVIRONMENT

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ABSTRACT

In today's scenario, a good teacher helps the students to view the real world as it is and make a remarkable impact on students' life from their classroom learning. Teachers play a crucial role in the lives of their students by providing guidance, promoting creativity & curiosity, and developing knowledge and abilities. Teachers guide the students to meet their academic goals and mould the student's ethical behavior toward success in their future career. So, the efficiency of teachers in the classroom is important in helping the students to achieve their goals. In modern times, feedback systems are essential to enhance the teaching and learning process. The feedback system provided to students supports in the improvement of the teacher's performance, methods, and way of teaching. In this paper, the students feedback on faculty performance is analysed under neutrosophic environment and the outstanding faculty is decided with the help of decision makers by ranking method and applying hybrid score accuracy functions of single-valued neutrosophic numbers.

Keywords: Neutrosophic set, Neutrosophic number, Score function, Accuracy function and Hybrid score function.

INTRODUCTION

Neutrosophic set theory was first introduced by Smarandache (2005) in order to represent uncertainty which arises in decision making process in the real world.

Single-valued neutrosophic set is an instance of neutrosophic set which represents the values in the interval [0,1]. The single valued neutrosophic set is a generalization of intuitionistic fuzzy set, where intuitionistic fuzzy set is a generalization fuzzy set. Fuzzy set is defined with respect to truth membership T only, the intuitionistic fuzzy set is defined with respect to truth membership T falsity membership F only, but the neutrosophic set is defined with respect to truth membership T, falsity membership F and indeterminacy membership I. So, single-valued neutrosophic sets are used to represent the uncertain situation, imprecise, incomplete, and inconsistent information's found in the real world.

The entropy-based grey relational analysis method was introduced by Biswas *et al.,* (2014) for multi-criteria decision making (MCDM) with a single-valued neutrosophic set. The fuzzy sets theory introduced by Zadeh (1965) deals with the difficulties involving uncertainty. In MCDM problems imprecision can be modeled by fuzzy set theory. Pramanik and Mukhopadhya (2011) executed an intuitionistic fuzzy MCDM strategy for teachers' selection according to the grey relational analysis. Rani *et al.,* (2021) applied single-valued neutrosophic critic multimoora framework for the multi-criteria food waste treatment method. Mei *et al.,* (2023) discussed multi-criteria group decision-making method based on an improved single-valued neutrosophic hamacher weighted average operator and grey relational analysis.

The essential component of decision-making is indeterminacy. The generalization of intuitionistic fuzzy sets which are neutrosophic sets are incorporated in the decision making process with uncertainty. MCDM aggregates each decision-maker's individual judgement to form a group decision matrix and identifies a group adequate solution that is most needed by the decision-makers. Ye (2013) initiated the correlation coefficient of SVNS's for single-valued neutrosophic multi-criteria decision-making problems. In the decision making process, ranking the order of alternatives yields the effective result. The objective of this paper is to develop neutrosophic multi-criteria group decision-making model based on hybrid score - accuracy functions with known weights under neutrosophic environment. New weighting technique is introduced to find out the weight of each criteria which helps to determine the best solution from the available facilities. The researchers have collected the real data from students feedback on faculty performance in a reputed Institution from Virudhunagar. Our problem is to decide the outstanding faculty with a help of decision makers by ranking method and applying hybrid score accuracy functions of single-valued neutrosophic numbers.

In section 2, under preliminaries, some fundamental definitions on neutrosophic sets and some operational definitions of multi-criteria group decision making method are discussed and an algorithm is applied to analyse the outstanding faculty in single-valued neutrosophic environment. In section 3, multi-criteria group decision making problem based on single-valued neutrosophic environment is developed.

PRELIMINARIES

In this chapter, Preliminaries, some fundamental definitions in [4] and [1] are discussed.

Definition 2.1 [4]: Let X be a non-empty set. A neutrosophic set A in X is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A, $T_A(x), I_A(x), F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$.

That is T_A: X→$]^-0,1^+[$, I_A: X→$]^-0,1^+[$, F_A: X→$]^-0,1^+[$

There is no restriction on the sum of $T_A(x)$, $I_A(x), F_A(x),$ so $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2.2 [4]: Let X be a non-empty set. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function T_A, falsity-membership functionF_A, and indeterminacy-membership function I_A. For each point x in X, T_A(x),I_A(x),F_A(x) ∈ [0,1]. A SVNS A can be written as, A= {(x, T_A (x), I_A (x), F_A (x))}: x∈X, T_A(x), I_A(x), F_A(x) ∈ [0,1].

Definition 2.3 [4]: The complement of a SVNS. A is denoted by A^C and defined by A^C= {(x, 1-T_A(x), 1-I_A(x), 1-F_A(x)) : x∈X.

Relations between two SVNSs:

1. A SVNS A is contained in the other SVNS B (A ⊆ B) if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$ and $F_A(x) \geq F_B(x)$, for all x∈X.
2. Two SVNSs A and B are equal (A=B) if and only if A ⊆ B and B ⊇ A.

Definition 2.4 [4] : For a single valued neutrosophic set (SVNS) A={(x, T_A(x),I_A(x), F_A(x))}: x∈X, T_A(x), I_A(x), F_A(x) ∈ [0,1] in X, the triplets (T_A(x), I_A(x), F_A(x)) is called single valued neutrosophic number (SVNN),which is the fundamental element of a SVNS A.

Definition 2.5 [1]: Let $\alpha = (T(\alpha), I(\alpha), F(\alpha))$ be a single valued neutrosophic number (SVNN). Then the score function and the accuracy function of the SVNN α can be represented respectively, as follows: $s(\alpha) = (1 + T(\alpha) - I(\alpha)) / 2$, and $s(\alpha) \in [0,1]$ $h(\alpha) = (2 + T(\alpha) - I(\alpha) - F(\alpha)) / 3$ and $h(\alpha) \in [0,1]$.

Definition 2.7 [1] : The hybrid score-accuracy matrix $\mathcal{H}^k = (\mathcal{H}_{ij}^k)_{m\times n}$ (k=1,2,…,t; i=1,2,…,m; j=1,2,…,n) is obtained from the decision matrix $D_k = (A_{ij}^k)_{m\times n}$ by the following formula:

$\mathcal{H}_{ij}^k = \frac{1}{2}(1 + T_{ij}^k - I_{ij}^k) + \frac{1}{3}(T_{ij}^k + 1 - F_{ij}^k + 1 - I_{ij}^k)$.

Definition 2.8 [1] : From the hybrid score-accuracy matrix, the average matrix $\mathcal{H}^A = (\mathcal{H}_{ij}^A)_{m\times n}$ (k=1,2..t; i=1,2…m; j=1,2,..n) is calculated by $\mathcal{H}_{ij}^A = \frac{1}{t}\sum_{k=1}^{t}\mathcal{H}_{ij}^k$

The collective correlation coefficient between $\mathcal{H}^k$ (k=1,2,..,t) and H^A is given as follows:

$$C_k = \sum_{i=1}^{m} \frac{\sum_{j=1}^{n}\mathcal{H}_{ij}^k\,\mathcal{H}_{ij}^A}{\sqrt{\sum_{j=1}^{n}(\mathcal{H}_{ij}^k)^2}\sqrt{\sum_{j=1}^{n}(\mathcal{H}_{ij}^A)^2}}$$

Definition 2.9 [1]: Weight model for decision makers can be defined as: $\gamma_k = \frac{C_k}{\sum_{k=1}^{t} C_k}$

Where $0 \le \gamma_k \le 1$, $\sum_{k=1}^{t}\gamma_k = 1$ for k=1,2,…,t

Definition 2.10 [1]: For the weight vector $\gamma = (\gamma_1, \gamma_2, .., \gamma_k)^T$ of decision makers obtained from equation, we accumulate all individual hybrid score-accuracy matrices of $\mathcal{H}^k = (\mathcal{H}_{ij}^k)_{m\times n}$ (k=1,2,..,t; i=1,2,..m; j=1,2,..,n) into a collective hybrid score-accuracy matrix $\mathcal{H} = (\mathcal{H}_{ij})_{m\times n}$ by the following formula :$\mathcal{H}_{ij} = \sum_{k=1}^{t}\gamma_k\mathcal{H}_{ij}^k$

Definition 2.11 [1]: The weight of criteria for this model has to be decided by the experts or committee of decision makers according to their requirements which is denoted by w_j, (j=number of criteria). The weight of criteria is based on the significance of the importance of the criteria and the total weight is always one.

Definition 2.12 [1]: From the collective hybrid score-accuracy matrix the weighted hybrid score-accuracy matrix $\mathcal{H}^W = (\mathcal{H}_{ij}^W)_{m\times n}$ (w=1,2,..t: i=1,2,..,m: j=1,2,..,n) is calculated by

$\mathcal{H}_{ij}^W = w_j\mathcal{H}_{ij}$.

Definition 2.13 [1] : To rank the alternatives, one can sum all values in each row of weighted hybrid score-accuracy matrix and find the overall weighted score-accuracy value of each alternative A_i (i=1,2,..m), $R(A_i) = \sum_{j=1}^{n} \mathcal{H}_{ij}^{W}$.

ALGORITHM

Step 1: Compute hybrid-score accuracy matrix for D_1 and D_2 .

Step 2: Compute average matrix $\mathcal{H}^A$ from hybrid score-accuracy matrix $\mathcal{H}^k$ where (k=1,2)

Step 3: Determine decision maker's weight γ_k by using collective correlation coefficient between hybrid score-accuracy matrix $\mathcal{H}^k$ and average matrix $\mathcal{H}^A$

Step 4: Evaluate collective hybrid score-accuracy matrix $\mathcal{H}$ by using decision maker's weights.

Step 5: Compute weights of criteria by using swing weighting Technique.

Step 6: Compute weighted hybrid score-accuracy matrix $\mathcal{H}^W$.

Step 7: Calculate overall weighted hybrid score-accuracy values R(F_i) for each faculty F_i $(i = 1,2,3,4)$

Step 8: Identity the outstanding faculty by ranking them in descending order.

Multi Criteria Group Decision Making Problem Based on Single Valued Neutrosophic Environment

Now-a-days, teachers play an important role in the improvement of students community within and outside the institution. Students are very concerned about their teachers in various aspects. So, students feedback system helps in the enhancement of teaching and learning process. It also provides the ability to improve the teacher's performance and teaching techniques. The researchers have collected feedback data on faculty performance from the reputed institution through Google forms from two group of students D_1 and D_2 in Virudhunagar. Our objective is to determine the outstanding faculty based on the following criteria: C_1 – Simplicity in presentation, C_2 – Being on time, C_3 – Encouragement, C_4 – Finishing the Course syllabus, C_5 – Explanation of difficulties ,C_6 – Personalised care, C_7 – Carrying out tests, C_8 – Attention to students, C_9 – Remaining neutral, C_{10} – Application of ICT devices.

In our problem, the researchers have chosen four faculty members namely F_1, F_2, F_3, F_4 evaluated by two Group of students D_1 and D_2 under ten criteria $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}$. The outstanding faculty $F_i (i = 1,2,3,4)$ is evaluated by the form of single valued neutrosophic numbers. Thus the two single valued neutrosophic decision matrix can be obtained from the two group of students D_1 and D_2 expressed respectively as follows.

Table shows Single-valued neutrosophic decision matrix for D_1 :

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
F_1	(0.7,0.3, 0.0)	(0.6,0.4, 0.0)	(0.6,0.3, 0.0)	(0.6,0.4, 0.0)	(0.5,0.5, 0.1)	(0.5,0.4, 0.1)	(0.5,0.4, 0.1)	(0.6,0.3, 0.1)	(0.5,0.4, 0.0)	(0.5,0.4, 0.0)
F_2	(0.7,0.3, 0.0)	(0.5,0.4, 0.0)	(0.7,0.4, 0.0)	(0.7,0.3, 0.0)	(0.5,0.4, 0.1)	(0.6,0.4, 0.0)	(0.6,0.4, 0.0)	(0.6,0.3, 0.0)	(0.6,0.4, 0.0)	(0.5,0.4, 0.0)
F_3	(0.6,0.4, 0.0)	(0.6,0.4, 0.0)	(0.4,0.5, 0.1)	(0.4,0.5, 0.2)	(0.5,0.4, 0.1)	(0.5,0.4, 0.2)	(0.4,0.5, 0.0)	(0.4,0.4, 0.2)	(0.4,0.5, 0.0)	(0.4,0.5, 0.0)
F_4	(0.6,0.4, 0.0)	(0.5,0.4, 0.1)	(0.4,0.5, 0.1)	(0.4,0.5, 0.1)	(0.5,0.5, 0.0)	(0.4,0.5, 0.1)	(0.4,0.5, 0.1)	(0.4,0.5, 0.1)	(0.4,0.5, 0.1)	(0.3,0.6, 0.0)

Table shows Single-valued neutrosophic decision matrix for D_2 :

D_2	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
F_1	(0.5,0.5, 0.0)	(0.6,0.4, 0.0)	(0.8,0.3, 0.0)	(0.5,0.5, 0.0)	(0.4,0.6, 0.0)	(0.5,0.4, 0.1)	(0.4,0.6, 0.0)	(0.4,0.5, 0.0)	(0.4,0.6, 0.0)	(0.4,0.5, 0.0)
F_2	(0.8,0.2, 0.0)	(0.6,0.4, 0.0)	(0.8,0.2, 0.0)	(0.7,0.3, 0.0)	(0.7,0.3, 0.0)	(0.6,0.4, 0.0)	(0.5,0.5, 0.0)	(0.5,0.5, 0.0)	(0.5,0.5, 0.0)	(0.3,0.7, 0.0)
F_3	(0.6,0.4, 0.0)	(0.6,0.4, 0.0)	(0.4,0.6, 0.0)	(0.4,0.6, 0.0)	(0.6,0.4, 0.0)	(0.6,0.4, 0.0)	(0.5,0.5, 0.0)	(0.6,0.4, 0.0)	(0.5,0,5, 0,0)	(0.4,0.6, 0.0)
F_4	(0.7,0.3, 0.0)	(0.8,0.2, 0.0)	(0.6,0.4, 0.0)	(0.5,0.5, 0.0)	(0.6,0.3, 0.0)	(0.6,0.4, 0.0)	(0.3,0.6, 0.0)	(0.5,0.5, 0.0)	(0.5,0.5, 0.0)	(0.6,0.4, 0.0)

The two group of decision makers are D_1 and D_2. The hybrid score-accuracy matrix for decision maker D_1:

$$\mathcal{H}_{ij}^{k} = \frac{1}{2}(1+ T_{ij}^{k} - I_{ij}^{k}) + \frac{1}{3}(T_{ij}^{k} + 1 - F_{ij}^{k} + 1 - I_{ij}^{k}), \ k=1$$

$$\mathcal{H}_{11}^{1} = \frac{1}{2}(1+T_{11}^{1} - I_{11}^{1}) + \frac{1}{3}(T_{11}^{1} + 1 - F_{11}^{1} + 1 - I_{11}^{1})$$

$$= \frac{1}{2}(1+0.7 - 0.3) + \frac{1}{3}(0.7 + 1 - 0.0 + 1 - 0.3) = 0.2 + 0.8 = 1$$

$$\mathcal{H}_{12}^{1} = \frac{1}{2}(1+ T_{12}^{1} - I_{12}^{1}) + \frac{1}{3}(T_{12}^{1} + 1 - F_{12}^{1} + 1 - I_{12}^{1})$$

$$= \frac{1}{2}(1+ 0.6 - 0.4) + \frac{1}{3}(0.6 + 1 - 0.0 + 1 - 0.4) = 0.6 + 0.7333 = 1.3333$$

$$\mathcal{H}_{13}^{1} = \frac{1}{2}(1 + T_{13}^{1} - I_{13}^{1}) + \frac{1}{3}(T_{13}^{1} + 1 - F_{13}^{1} + 1 - I_{13}^{1})$$

$= \frac{1}{2}(1+ 0.6 – 0.3) + \frac{1}{3}(0.6 + 1 – 0.0 + 1 – 0.3) = 1.4166$

$\mathcal{H}_{14}^{1} = \frac{1}{2}(1 + T_{14}^{1} - I_{14}^{1}) + \frac{1}{3}(T_{14}^{1} + 1 - F_{14}^{1} + 1 - I_{14}^{1})$

$= \frac{1}{2}(1 + 0.6 – 0.4) + \frac{1}{3}(0.6 + 1 – 0.0 + 1 – 0.4) = 1.3333$

$\mathcal{H}_{15}^{1} = \frac{1}{2}(1 + T_{15}^{1} - I_{15}^{1}) + \frac{1}{3}(T_{15}^{1} + 1 - F_{15}^{1} + 1 - I_{15}^{1})$

$= \frac{1}{2}(1 + 0.5 – 0.5) + \frac{1}{3}(0.5 + 1 – 0.1 + 1 – 0.5) = 1.1333$

$\mathcal{H}_{16}^{1} = \frac{1}{2}(1 + T_{16}^{1} - I_{16}^{1}) + \frac{1}{3}(T_{16}^{1} + 1 - F_{16}^{1} + 1 - I_{16}^{1})$

$= \frac{1}{2}(1 + 0.5 - 0.4) + \frac{1}{3}(0.5 + 1 – 0.1 + 1 – 0.4) = 1.2166$

$\mathcal{H}_{17}^{1} = \frac{1}{2}(1 + T_{17}^{1} - I_{17}^{1}) + \frac{1}{3}(T_{17}^{1} + 1 – F_{17}^{1} + 1 - I_{17}^{1})$

$= \frac{1}{2}(1 + 0.5 – 0.4) + \frac{1}{3}(0.5 + 1 – 0.1 + 1 – 0.4) = 1.2166$

$\mathcal{H}_{18}^{1} = \frac{1}{2}(1 + T_{18}^{1} - I_{18}^{1}) + \frac{1}{3}(T_{18}^{1} + 1 - F_{18}^{1} + 1 - I_{18}^{1})$

$= \frac{1}{2}(1 + 0.6 – 0.3) + \frac{1}{3}(0.6 + 1 – 0.1 + 1 - 0.3) = 1.3833$

$\mathcal{H}_{19}^{1} = \frac{1}{2}(1 + T_{19}^{1} - I_{19}^{1}) + \frac{1}{3}(T_{19}^{1} + 1 - F_{19}^{1} + 1 - I_{19}^{1})$

$= \frac{1}{2}(1 + 0.5 – 0.4) + \frac{1}{3}(0.5 + 1 – 0.0 + 1 – 0.4) = 1.25$

$\mathcal{H}_{110}^{1} = \frac{1}{2}(1 + T_{110}^{1} - I_{110}^{1}) + \frac{1}{3}(T_{110}^{1} + 1 - F_{110}^{1} + 1 - I_{110}^{1})$

$= \frac{1}{2}(1 + 0.5 – 0.4) + \frac{1}{3}(0.5 + 1 - 0.0 + 1 – 0.4) = 1.25$

$\mathcal{H}_{21}^{1} = \frac{1}{2}(1 + T_{21}^{1} - I_{21}^{1}) + \frac{1}{3}(T_{21}^{1} + 1 - F_{21}^{1} + 1 - I_{21}^{1})$

$= \frac{1}{2}(1 + 0.7 – 0.3) + \frac{1}{2}(0.7 + 1 – 0.0 + 1 – 0.3) = 1.5$

$\mathcal{H}_{22}^{1} = \frac{1}{2}(1 + T_{22}^{1} - I_{22}^{1}) + \frac{1}{3}(T_{22}^{1} + 1 - F_{22}^{1} + 1 - I_{22}^{1})$

$= \frac{1}{2}(1 + 0.5 – 0.4) + \frac{1}{3}(0.5 + 1 – 0.0 + 1 – 0.4) = 1.25$

$\mathcal{H}_{23}^{1} = \frac{1}{2}(1 + T_{23}^{1} - I_{23}^{1}) + \frac{1}{3}(T_{23}^{1} + 1 - F_{23}^{1} + 1 - I_{23}^{1})$

$= \frac{1}{2}(1 + 0.7 – 0.4) + \frac{1}{3}(0.7 + 1 – 0.0 + 1 – 0.4) = 1.4166$

$\mathcal{H}_{24}^{1} = \frac{1}{2}(1 + T_{24}^{1} - I_{24}^{1}) + \frac{1}{3}(T_{24}^{1} + 1 - F_{24}^{1} + 1 - I_{24}^{1})$

$= \frac{1}{2}(1 + 0.7 – 0.3) + \frac{1}{3}(0.7 + 1 – 0.0 + 1 – 0.3) = 1.5$

$\mathcal{H}_{25}^{1} = \frac{1}{2}(1 + T_{25}^{1} - I_{25}^{1}) + \frac{1}{3}(T_{25}^{1} + 1 - F_{25}^{1} + 1 - I_{25}^{1})$

$= \frac{1}{2}(1 + 0.5 – 0.4) + \frac{1}{2}(0.5 + 1 – 0.1 + 1 – 0.4) = 1.2166$

$\mathcal{H}_{26}^{1} = \frac{1}{2}(1 + T_{26}^{1} - I_{26}^{1}) + \frac{1}{3}(T_{26}^{1} + 1 - F_{26}^{1} + 1 - I_{26}^{1})$

$= \frac{1}{2}(1+ 0.6 – 0.4) + \frac{1}{3}(0.6 + 1 – 0.0 + 1 – 0.4) = 1.3333$

$\mathcal{H}_{27}^{1} = \frac{1}{2}(1+ T_{27}^{1} - I_{27}^{1}) + \frac{1}{3}(T_{27}^{1} + 1 – F_{27}^{1} + 1 - I_{27}^{1})$

$= \frac{1}{2}(1 + 0.6 – 0.4) + \frac{1}{3}(0.6 + 1 – 0.0 + 1 – 0.4) = 1.3333$

$\mathcal{H}_{28}^{1} = \frac{1}{2}(1 + T_{28}^{1} - I_{28}^{1}) + \frac{1}{3}(T_{28}^{1} + 1 - F_{28}^{1} + 1 - I_{28}^{1})$

$= \frac{1}{2}(1 + 0.6 – 0.3) + \frac{1}{3}(0.6 + 1 – 0.0 + 1 – 0.3) = 1.4166$

$\mathcal{H}_{29}^{1} = \frac{1}{2}(1+ T_{29}^{1} - I_{29}^{1}) + \frac{1}{3}(T_{29}^{1} + 1 - F_{29}^{1} + 1 - I_{29}^{1})$

$= \frac{1}{2}(1 + 0.6 – 0.4) + (0.6 + 1 – 0.0 + 1 – 0.4) = 1.3333$

$\mathcal{H}_{210}^{1} = \frac{1}{2}(1 + T_{210}^{1} - I_{210}^{1}) + \frac{1}{3}(T_{210}^{1} + 1 - F_{210}^{1} + 1 - I_{210}^{1})$

$= \frac{1}{2}(1+ 0.5 – 0.4) + \frac{1}{3}(0.5 + 1 – 0.0 + 1 – 0.4) = 1.25$

$\mathcal{H}_{31}^{1} = \frac{1}{2}(1+ T_{31}^{1} - I_{31}^{1}) + \frac{1}{3}(T_{31}^{1} + 1 - F_{31}^{1} + 1 - I_{31}^{1})$

$= \frac{1}{2}(1 + 0.6 - 0.4) + \frac{1}{3}(0.6 + 1 – 0.0 + 1 – 0.4) = 1.3333$

$\mathcal{H}^1_{32} = \frac{1}{2}(1 + T^1_{32} - I^1_{32}) + \frac{1}{3}(T^1_{32} + 1 - F^1_{32} + 1 - I^1_{32})$

$= \frac{1}{2}(1 + 0.6 - 0.4) + \frac{1}{3}(0.6 + 1 – 0.0 + 1 – 0.4) = 1.3333$

$\mathcal{H}^1_{33} = \frac{1}{2}(1 + T^1_{33} - I^1_{33}) + \frac{1}{3}(T^1_{33} + 1 - F^1_{33} + 1 - I^1_{33})$

$= \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.1 + 1 – 0.5) = 1.05$

$\mathcal{H}^1_{34} = \frac{1}{2}(1 + T^1_{34} - I^1_{34}) + \frac{1}{3}(T^1_{34} + 1 - F^1_{34} + 1 - I^1_{34})$

$= \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.2 + 1 – 0.5) = 1.0166$

$\mathcal{H}^1_{35} = \frac{1}{2}(1 + T^1_{35} - I^1_{35}) + \frac{1}{3}(T^1_{35} + 1 - F^1_{35} + 1 - I^1_{35})$

$= \frac{1}{2}(1 + 0.5 - 0.4) + \frac{1}{3}(0.5 + 1 – 0.1 + 1 – 0.4) = 1.2166$

$\mathcal{H}^1_{36} = \frac{1}{2}(1 + T^1_{36} - I^1_{36}) + \frac{1}{3}(T^1_{36} + 1 - F^1_{36} + 1 - I^1_{36})$

$= \frac{1}{2}(1 + 0.5 - 0.4) + \frac{1}{3}(0.5 + 1 – 0.2 + 1 – 0.4) = 1.2833$

$\mathcal{H}^1_{37} = \frac{1}{2}(1 + H^1_{37} - I^1_{37}) + \frac{1}{3}(T^1_{37} + 1 - F^1_{37} + 1 - I^1_{37})$

$= \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.0 + 1 – 0.5) = 1.0833$

$\mathcal{H}^1_{38} = \frac{1}{2}(1 + T^1_{38} - I^1_{38}) + \frac{1}{3}(T^1_{38} + 1 - F^1_{38} + 1 - I^1_{38})$

$= \frac{1}{2}(1 + 0.4 - 0.5) + \frac{1}{3}(0.4 + 1 - 0.2 + 1 - 0.5) = 1.1$

$\mathcal{H}^1_{39} = \frac{1}{2}(1 + T^1_{39} - I^1_{39}) + \frac{1}{3}(T^1_{39} + 1 - F^1_{39} + 1 - I^1_{39})$

$= \frac{1}{2}(1 + 0.4 - 0.5) + \frac{1}{3}(0.4 + 1 - 0.0 + 1 - 0.5) = 1.0833$

$\mathcal{H}^1_{310} = \frac{1}{2}(1 + T^1_{310} - I^1_{310}) + \frac{1}{3}(T^1_{310} + 1 - F^1_{310} + 1 - I^1_{310})$

$= \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.0 + 1 – 0.5) = 1.0833$

$\mathcal{H}^1_{41} = \frac{1}{2}(1 + T^1_{41} - I^1_{41}) + \frac{1}{3}(T^1_{41} + 1 - F^1_{41} + 1 - I^1_{41})$

$= \frac{1}{2}(1 + 0.6 – 0.4) + \frac{1}{3}(0.6 + 1 – 0.0 + 1 – 0.4) = 1.3333$

$\mathcal{H}^1_{42} = \frac{1}{2}(1 + T^1_{42} - I^1_{42}) + \frac{1}{3}(T^1_{42} + 1 - F^1_{42} + 1 - I^1_{42})$

$= \frac{1}{2}(1 + 0.5 – 0.4) + \frac{1}{3}(0.5 + 1 – 0.1 + 1 – 0.4) = 1.2166$

$\mathcal{H}^1_{43} = \frac{1}{2}(1 + T^1_{43} - I^1_{43}) + \frac{1}{3}(T^1_{43} + 1 - F^1_{43} + 1 - I^1_{43})$

$= \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.1 + 1 – 0.5) = 1.05$

$\mathcal{H}^1_{44} = \frac{1}{2}(1 + T^1_{44} - I^1_{44}) + \frac{1}{3}(T^1_{44} + 1 - F^1_{44} + 1 - I^1_{44})$

$= \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.1 + 1 – 0.5) = 1.05$

$\mathcal{H}^1_{45} = \frac{1}{2}(1 + T^1_{45} - I^1_{45}) + \frac{1}{3}(T^1_{45} + 1 - F^1_{45} + 1 - I^1_{45})$

$= \frac{1}{2}(1 + 0.5 – 0.5) + \frac{1}{3}(0.5 + 1 – 0.0 + 1 – 0.5) = 1.1666$

$\mathcal{H}^1_{46} = \frac{1}{2}(1 + T^1_{46} - I^1_{46}) + \frac{1}{3}(T^1_{46} + 1 - F^1_{46} + 1 - I^1_{46})$

$= \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.1 + 1 – 0.5) = 1.05$

$\mathcal{H}^1_{47} = \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.1 + 1 – 0.5) = 1.05$

$\mathcal{H}^1_{48} = \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.1 + 1 – 0.5) = 1.05$

$\mathcal{H}^1_{49} = \frac{1}{2}(1 + 0.4 – 0.5) + \frac{1}{3}(0.4 + 1 – 0.1 + 1 – 0.5) = 1.05$

$\mathcal{H}^1_{410} = \frac{1}{2}(1 + 0.3 – 0.6) + \frac{1}{3}(0.3 + 1 – 0.0 + 1 – 0.6) = 0.9166$

Table shows the hybrid score-accuracy matrix for the decision maker D_1:

$\mathcal{H}^1$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
F_1	1	1.3333	1.4166	1.3333	1.1333	1.2166	1.2166	1.3833	1.25	1.25
F_2	1.5	1.25	1.4166	1.5	1.2166	1.3333	1.3333	1.4166	1.3333	1.25
F_3	1.3333	1.3333	1.05	1.0166	1.2166	1.2833	1.0833	1.1	1.0833	1.0833
F_4	1.3333	1.2166	1.05	1.05	1.1666	1.05	1.05	1.05	1.05	0.9166

Similarly, we compute hybrid score-accuracy the decision maker D_2:

Table shows the hybrid score-accuracy matrix for the decision maker D_2:

$\mathcal{H}^2$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
F_1	1.1666	1.3333	1.5833	1.1666	1	1.2166	1	1.0833	1	1.0833
F_2	1.6666	1.3333	1.6666	1.5	1.5	1.3333	1.1666	1.1666	1.1666	0.85
F_3	1.3333	1.3333	1	1	1.3333	1.3333	1.1666	1.3333	1.1666	1
F_4	1.5	1.6666	1.3333	1.1666	1.4166	1.3333	0.9166	1.1666	1.1666	1.3333

Compute the average matrix $\mathcal{H}^A$ from the hybrid score-accuracy matrices for the decision makers D_1 and D_2:

$\mathcal{H}^A_{11} = \frac{1}{2}\sum_{k=1}^{2}\mathcal{H}^k_{11} = \frac{1}{2}(\mathcal{H}^1_{11} + \mathcal{H}^2_{11}) = \frac{1}{2}$ (1+1.1666) = 1.0833

$\mathcal{H}^A_{12} = \frac{1}{2}(\mathcal{H}^1_{12} + \mathcal{H}^2_{12}) = \frac{1}{2}$(1.3333+1.3333) = 1.3333

$\mathcal{H}^A_{13} = \frac{1}{2}(\mathcal{H}^1_{13} + \mathcal{H}^2_{13}) = \frac{1}{2}$(1.4166+1.5833) = 1.49995

$\mathcal{H}^A_{14} = \frac{1}{2}(\mathcal{H}^1_{14} + \mathcal{H}^2_{14}) = \frac{1}{2}$(1.3333+ 1.4166) = 1.24995

$\mathcal{H}^A_{15} = \frac{1}{2}(\mathcal{H}^1_{15} + \mathcal{H}^2_{15}) = \frac{1}{2}$(1.1333+1) = 1.0667

$\mathcal{H}^A_{16} = \frac{1}{2}(\mathcal{H}^1_{16} + \mathcal{H}^2_{16}) = \frac{1}{2}$(1.2166+1.2166) = 1.2166

$\mathcal{H}^A_{17} = \frac{1}{2}(\mathcal{H}^1_{17} + \mathcal{H}^2_{17}) = \frac{1}{2}$(1.2166+1) = 1.1083

$\mathcal{H}^A_{18} = \frac{1}{2}(\mathcal{H}^1_{18} + \mathcal{H}^2_{18}) = \frac{1}{2}$(1.3833+ 1.0833) = 1.2333

$\mathcal{H}^A_{19} = \frac{1}{2}(\mathcal{H}^1_{19} + \mathcal{H}^2_{19}) = \frac{1}{2}$(1.25+ 1) = 1.125

$\mathcal{H}^A_{110} = \frac{1}{2}(\mathcal{H}^1_{110} + \mathcal{H}^2_{110}) = \frac{1}{2}$(1.25+1.0833) = 1.1667

$\mathcal{H}^A_{21} = \frac{1}{2}(\mathcal{H}^1_{21} + \mathcal{H}^2_{21}) = \frac{1}{2}$(1.5+1.6666) = 1.5833

$\mathcal{H}^A_{22} = \frac{1}{2}(\mathcal{H}^1_{22} + \mathcal{H}^2_{22}) = \frac{1}{2}$(1.25+1.3333) = 1.2917

$\mathcal{H}^A_{23} = \frac{1}{2}(\mathcal{H}^1_{23} + \mathcal{H}^2_{23}) = \frac{1}{2}$(1.4166+1.6666) = 1.5416

$\mathcal{H}^A_{24} = \frac{1}{2}(\mathcal{H}^1_{24} + \mathcal{H}^2_{24}) = \frac{1}{2}$(1.5+1.5) = 1.5

$\mathcal{H}^A_{25} = \frac{1}{2}(\mathcal{H}^1_{25} + \mathcal{H}^2_{25}) = \frac{1}{2}$(1.2166+1.5) =1.3583

$\mathcal{H}^A_{26} = \frac{1}{2}(\mathcal{H}^1_{26} + \mathcal{H}^2_{26} = \frac{1}{2}$(1.3333+1.3333) = 1.3333

$\mathcal{H}^A_{27} = \frac{1}{2}(\mathcal{H}^1_{27} + \mathcal{H}^2_{27}) = \frac{1}{2}$(1.3333+1.1666) =1.24995

$\mathcal{H}^A_{28} = \frac{1}{2}(\mathcal{H}^1_{28} + \mathcal{H}^2_{28}) = \frac{1}{2}$(1.4166+1.1666) = 1.2916

$\mathcal{H}^A_{29} = \frac{1}{2}(\mathcal{H}^1_{29} + \mathcal{H}^2_{29}) = \frac{1}{2}$(1.3333+1.1666)

= 1.2498

$\mathcal{H}_{210}^{A} = \frac{1}{2}(\mathcal{H}_{210}^{1} + \mathcal{H}_{210}^{2}) = \frac{1}{2}(1.25+0.85)$
= 1.05

$\mathcal{H}_{31}^{A} = \frac{1}{2}(\mathcal{H}_{31}^{1} + \mathcal{H}_{31}^{2}) = \frac{1}{2}(1.3333+1.3333)$
= 1.3333

$\mathcal{H}_{32}^{A} = \frac{1}{2}(\mathcal{H}_{32}^{1} + \mathcal{H}_{32}^{2}) = \frac{1}{2}(1.3333+1.3333)$
= 1.3333

$\mathcal{H}_{33}^{A} = \frac{1}{2}(\mathcal{H}_{33}^{1} + \mathcal{H}_{33}^{2}) = \frac{1}{2}(1.05+1)$
=1.025

$\mathcal{H}_{34}^{A} = \frac{1}{2}(\mathcal{H}_{34}^{1} + \mathcal{H}_{34}^{2}) = \frac{1}{2}(1.0166+1)$
= 1.0083

$\mathcal{H}_{35}^{A} = \frac{1}{2}(\mathcal{H}_{35}^{1} + \mathcal{H}_{35}^{2}) = \frac{1}{2}(1.2166+1.3333)$
= 1.27495

$\mathcal{H}_{36}^{A} = \frac{1}{2}(\mathcal{H}_{36}^{1} + \mathcal{H}_{36}^{2}) = \frac{1}{2}(1.2833+1.3333)$
= 1.3083

$\mathcal{H}_{37}^{A} = \frac{1}{2}(\mathcal{H}_{37}^{1} + \mathcal{H}_{37}^{2}) = \frac{1}{2}(1.0833+1.1666)$
= 1.12495

$\mathcal{H}_{38}^{A} = \frac{1}{2}(\mathcal{H}_{38}^{1} + \mathcal{H}_{38}^{2}) = \frac{1}{2}(1.1+1.3333)$
= 1.2167

$\mathcal{H}_{39}^{A} = \frac{1}{2}(\mathcal{H}_{39}^{1} + \mathcal{H}_{39}^{2}) = \frac{1}{2}(1.0833+1.1666)$
= 1.12495

$\mathcal{H}_{310}^{A} = \frac{1}{2}(\mathcal{H}_{310}^{1} + \mathcal{H}_{310}^{2}) = \frac{1}{2}(1.0833+1)$
= 1.0417

$\mathcal{H}_{41}^{A} = \frac{1}{2}(\mathcal{H}_{41}^{1} + \mathcal{H}_{41}^{2}) = \frac{1}{2}(1.3333+1.5)$
= 1.4167

$\mathcal{H}_{42}^{A} = \frac{1}{2}(\mathcal{H}_{42}^{1} + \mathcal{H}_{42}^{2}) = \frac{1}{2}(1.2166+1.6666)$
= 1.4416

$\mathcal{H}_{43}^{A} = \frac{1}{2}(\mathcal{H}_{43}^{1} + \mathcal{H}_{43}^{2}) = \frac{1}{2}(1.05+1.3333)$
= 1.1917

$\mathcal{H}_{44}^{A} = \frac{1}{2}(\mathcal{H}_{44}^{1} + \mathcal{H}_{44}^{2}) = \frac{1}{2}(1.05+1.1666)$
= 1.1083

$\mathcal{H}_{45}^{A} = \frac{1}{2}(\mathcal{H}_{45}^{1} + \mathcal{H}_{45}^{2}) = \frac{1}{2}(1.1666+1.4166)$
= 1.2916

$\mathcal{H}_{46}^{A} = \frac{1}{2}(\mathcal{H}_{46}^{1} + \mathcal{H}_{46}^{2}) = \frac{1}{2}(1.05+1.3333)$
= 1.1917

$\mathcal{H}_{47}^{\boldsymbol{A}} = \frac{1}{2}(\mathcal{H}_{47}^{\boldsymbol{1}} + \mathcal{H}_{47}^{\boldsymbol{2}}) = \frac{1}{2}(1.05+0.9166)$
= 0.9833

$\mathcal{H}_{48}^{\boldsymbol{A}} = \frac{1}{2}(\mathcal{H}_{48}^{\boldsymbol{1}} + \mathcal{H}_{48}^{\boldsymbol{2}}) = \frac{1}{2}(1.05+1.1666)$
= 1.1083

$\mathcal{H}_{49}^{\boldsymbol{A}} = \frac{1}{2}(\mathcal{H}_{49}^{\boldsymbol{1}} + \mathcal{H}_{49}^{\boldsymbol{2}}) = \frac{1}{2}(1.05+1.1666)$
= 1.1083

$\mathcal{H}_{410}^{\boldsymbol{A}} = \frac{1}{2}(\mathcal{H}_{410}^{\boldsymbol{1}} + \mathcal{H}_{410}^{\boldsymbol{2}}) = \frac{1}{2}(0.9166+1.3333)$
=1.12495

Table shows the Average matrix $\mathcal{H}^{A}$:

$\boldsymbol{\mathcal{H}^{A}}$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
F_1	1.0833	1.3333	1.49995	1.24995	1.0667	1.2166	1.1083	1.2333	1.125	1.1667
F_2	1.5833	1.2917	1.5416	1.5	1.3583	1.3333	1.24995	1.2916	1.2498	1.05
F_3	1.3333	1.3333	1.025	1.0083	1.27495	1.3083	1.12495	1.12167	1.12495	1.0417
F_4	1.4167	1.4416	1.1917	1.1083	1.2916	1.1917	0.9833	1.1083	1.1083	1.12495

Compute the weights of two decision makers by using the formula γ_k:

$\gamma_k = \frac{C_k}{\sum_{k=1}^{t} C_k}$, i=1,2,3,4 and j=1,2,…,10,

k=1,2 where $C_k = \sum_{i=1}^{m} \frac{\sum_{j=1}^{n} \mathcal{H}_{ij}^{k} \mathcal{H}_{ij}^{A}}{\sqrt{\sum_{j=1}^{n} (\mathcal{H}_{ij}^{k})^2}\sqrt{\sum_{j=1}^{n} (\mathcal{H}_{ij}^{A})^2}}$

Put k=1, we get

$C_1 = \sum_{i=1}^{4} \frac{\sum_{j=1}^{10} \mathcal{H}_{ij}^{1} \mathcal{H}_{ij}^{A}}{\sqrt{\sum_{j=1}^{10} (\mathcal{H}_{ij}^{1})^2}\sqrt{\sum_{j=1}^{10} (\mathcal{H}_{ij}^{A})^2}}$

$\sum_{i=1}^{4}(\sum_{j=1}^{10} \mathcal{H}_{ij}^{1} \mathcal{H}_{ij}^{A}) = [\mathcal{H}_{11}^{1}\mathcal{H}_{11}^{A} + \mathcal{H}_{12}^{1}\mathcal{H}_{12}^{A} + \mathcal{H}_{13}^{1}\mathcal{H}_{13}^{A} + \mathcal{H}_{14}^{1}\mathcal{H}_{14}^{A} +$

$\mathcal{H}^1_{15}\mathcal{H}^A_{15} + \mathcal{H}^1_{16}\mathcal{H}^A_{16} + \mathcal{H}^1_{17}\mathcal{H}^A_{17} + \mathcal{H}^1_{18}\mathcal{H}^A_{18} + \mathcal{H}^1_{19}\mathcal{H}^A_{19} + \mathcal{H}^1_{110}\mathcal{H}^A_{110}] + [\mathcal{H}^1_{21}\mathcal{H}^A_{21} + \mathcal{H}^1_{22}\mathcal{H}^A_{22} + \mathcal{H}^1_{23}\mathcal{H}^A_{23} + \mathcal{H}^1_{24}\mathcal{H}^A_{24} + \mathcal{H}^1_{25}\mathcal{H}^A_{25} + \mathcal{H}^1_{26}\mathcal{H}^A_{26} + \mathcal{H}^1_{27}\mathcal{H}^A_{27} + \mathcal{H}^1_{28}\mathcal{H}^A_{28} + \mathcal{H}^1_{29}\mathcal{H}^A_{29} + \mathcal{H}^1_{210}\mathcal{H}^A_{210}] + [\mathcal{H}^1_{31}\mathcal{H}^A_{31} + \mathcal{H}^1_{32}\mathcal{H}^A_{32} + \mathcal{H}^1_{33}\mathcal{H}^A_{33} + \mathcal{H}^1_{34}\mathcal{H}^A_{34} + \mathcal{H}^1_{35}\mathcal{H}^A_{35} + \mathcal{H}^1_{36}\mathcal{H}^A_{36} + \mathcal{H}^1_{37}\mathcal{H}^A_{37} + \mathcal{H}^1_{38}\mathcal{H}^A_{38} + \mathcal{H}^1_{39}\mathcal{H}^A_{39} + \mathcal{H}^1_{310}\mathcal{H}^A_{310}] + [\mathcal{H}^1_{41}\mathcal{H}^A_{41} + \mathcal{H}^1_{42}\mathcal{H}^A_{42} + \mathcal{H}^1_{43}\mathcal{H}^A_{43} + \mathcal{H}^1_{44}\mathcal{H}^A_{44} + \mathcal{H}^1_{45}\mathcal{H}^A_{45} + \mathcal{H}^1_{46}\mathcal{H}^A_{46} + \mathcal{H}^1_{47}\mathcal{H}^A_{47} + \mathcal{H}^1_{48}\mathcal{H}^A_{48} + \mathcal{H}^1_{49}\mathcal{H}^A_{49} + \mathcal{H}^1_{410}\mathcal{H}^A_{410}]$

= [(1)(1.0833)+(1.3333)(1.3333)+(1.4166)(1.49995)+(1.3333)(1.24995)+(1.1333)(1.0667)+(1.2166)(1.2166)+(1.2166)(1.1083)+(1.3833)(1.2333)+(1.25)(1.125)+(1.25)(1.1667)]+[(1.5)(1.5833)(1.25)(1.2917)+(1.4166)(1.5416)+(1.5)(1.5)+(1.2166)(1.3583) +(1.3333) (1.3333+(1.3333)(1.24995)+(1.4166)(1.2916)+(1.3333)(1.2498)+(1.25)(1.05)]+[(1.3333) (1.3333)+(1.3333)(1.3333)+(1.05)(1.025)+(1.0166)(1.0083)+(1.2166)(1.27495)+(1.2833)(1.3083)+(1.0833)(1.12495)+(1.1)(1.2167)+(1.0833)(1.12495)+(1.0833)(1.0417)+[(1.3333) (1.4167)+(1.2166)(1.4416)+(1.05)(1.1917)+(1.05)(1.1083)+(1.1666)(1.2916)+(1.05)(1.1917) +(1.05)(0.9833)+(1.05)(1.1083)+(1.05)(1.1083)+(0.9166)(1.12495)]

$$\sum_{i=1}^{4}\left(\sum_{j=1}^{10}\mathcal{H}^1_{ij}\,\mathcal{H}^A_{ij}\right) = 60.58678701$$

Now,

$$\sum_{i=1}^{4}\sqrt{\sum_{j=1}^{10}(\mathcal{H}^1_{ij})^2}\sqrt{\sum_{j=1}^{n}(\mathcal{H}^A_{ij})^2}$$

$$= \left(\sqrt{(\mathcal{H}^1_{11})^2 + (\mathcal{H}^1_{12})^2 + (\mathcal{H}^1_{13})^2 + (\mathcal{H}^1_{14})^2 + (\mathcal{H}^1_{15})^2 + (\mathcal{H}^1_{16})^2 + (\mathcal{H}^1_{17})^2 + (\mathcal{H}^1_{18})^2 + (\mathcal{H}^1_{19})^2 + (\mathcal{H}^1_{110})^2}\right)$$
$$\left(\sqrt{(\mathcal{H}^A_{11})^2 + (\mathcal{H}^A_{12})^2 + (\mathcal{H}^A_{13})^2 + (\mathcal{H}^A_{14})^2 + (\mathcal{H}^A_{15})^2 + (\mathcal{H}^A_{16})^2 + (\mathcal{H}^A_{17})^2 + (\mathcal{H}^A_{18})^2 + (\mathcal{H}^A_{19})^2 + (\mathcal{H}^A_{110})^2}\right)$$
$$+ \left(\sqrt{(\mathcal{H}^1_{21})^2 + (\mathcal{H}^1_{22})^2 + (\mathcal{H}^1_{23})^2 + (\mathcal{H}^1_{24})^2 + (\mathcal{H}^1_{25})^2 + (\mathcal{H}^1_{26})^2 + (\mathcal{H}^1_{27})^2 + (\mathcal{H}^1_{28})^2 + (\mathcal{H}^1_{29})^2 + (\mathcal{H}^1_{210})^2}\right)$$
$$\left(\sqrt{(\mathcal{H}^A_{21})^2 + (\mathcal{H}^A_{22})^2 + (\mathcal{H}^A_{23})^2 + (\mathcal{H}^A_{24})^2 + (\mathcal{H}^A_{25})^2 + (\mathcal{H}^A_{26})^2 + (\mathcal{H}^A_{27})^2 + (\mathcal{H}^A_{28})^2 + (\mathcal{H}^A_{29})^2 + (\mathcal{H}^A_{210})^2}\right)$$
$$+ \left(\sqrt{(\mathcal{H}^1_{31})^2 + (\mathcal{H}^1_{32})^2 + (\mathcal{H}^1_{33})^2 + (\mathcal{H}^1_{34})^2 + (\mathcal{H}^1_{35})^2 + (\mathcal{H}^1_{36})^2 + (\mathcal{H}^1_{37})^2 + (\mathcal{H}^1_{38})^2 + (\mathcal{H}^1_{39})^2 + (\mathcal{H}^1_{310})^2}\right)$$
$$\left(\sqrt{(\mathcal{H}^A_{31})^2 + (\mathcal{H}^A_{32})^2 + (\mathcal{H}^A_{33})^2 + (\mathcal{H}^A_{34})^2 + (\mathcal{H}^A_{35})^2 + (\mathcal{H}^A_{36})^2 + (\mathcal{H}^A_{37})^2 + (\mathcal{H}^A_{38})^2 + (\mathcal{H}^A_{39})^2 + (\mathcal{H}^A_{310})^2}\right)$$

$$+\left(\sqrt{(\mathcal{H}_{41}^{1})^2+(\mathcal{H}_{42}^{1})^2+(\mathcal{H}_{43}^{1})^2+(\mathcal{H}_{44}^{1})^2+(\mathcal{H}_{45}^{1})^2+(\mathcal{H}_{46}^{1})^2+(\mathcal{H}_{47}^{1})^2+(\mathcal{H}_{48}^{1})^2+(\mathcal{H}_{49}^{1})^2+(\mathcal{H}_{410}^{1})^2}\right)$$

$$\left(\sqrt{(\mathcal{H}_{41}^{A})^2+(\mathcal{H}_{42}^{A})^2+(\mathcal{H}_{43}^{A})^2+(\mathcal{H}_{44}^{A})^2+(\mathcal{H}_{45}^{A})^2+(\mathcal{H}_{46}^{A})^2+(\mathcal{H}_{47}^{A})^2+(\mathcal{H}_{48}^{A})^2+(\mathcal{H}_{49}^{A})^2+(\mathcal{H}_{410}^{A})^2}\right)$$

$$=\left(\sqrt{(1)^2+(1.3333)^2+(1.4166)^2+(1.3333)^2+(1.1333)^2+(1.2166)^2+(1.2166)^2+(1.3833)^2+(1.25)^2+(1.25)^2}\right)$$

$$\left(\sqrt{(1.0833)^2+(1.3333)^2+(1.49995)^2+(1.24995)^2+(1.0667)^2+(1.2166)^2+(1.1083)^2+(1.2333)^2+(1.125)^2+(1.1667)^2}\right)$$

$$+\left(\sqrt{(1.5)^2+(1.25)^2+(1.4166)^2+(1.5)^2+(1.2166)^2+(1.3333)^2+(1.3333)^2+(1.4166)^2+(1.3333)^2+(1.25)^2}\right)$$

$$\left(\sqrt{(1.5833)^2+(1.2917)^2+(1.5416)^2+(1.5)^2+(1.3583)^2+(1.3333)^2+(1.24995)^2+(1.2916)^2+(1.2498)^2+(1.05)^2}\right)$$

$$+\left(\sqrt{(1.3333)^2+(1.3333)^2+(1.05)^2+(1.0166)^2+(1.2166)^2+(1.2833)^2+(1.0833)^2+(1.1)^2+(1.0833)^2+(1.0833)^2}\right)$$

$$\left(\sqrt{(1.3333)^2+(1.3333)^2+(1.025)^2+(1.0083)^2+(1.27495)^2+(1.3083)^2+(1.12495)^2+(1.2167)^2+(1.12495)^2+(1.0417)^2}\right)$$

$$+\left(\sqrt{(1.3333)^2+(1.2166)^2+(1.05)^2+(1.05)^2+(1.1666)^2+(1.05)^2+(1.05)^2+(1.05)^2+(1.05)^2+(0.9166)^2}\right)$$

$$\left(\sqrt{(1.4167)^2+(1.4416)^2+(1.1917)^2+(1.1083)^2+(1.2916)^2+(1.1917)^2+(0.9833)^2+(1.1083)^2+(1.1083)^2+(1.12495)^2}\right)$$

= (3.980609531)(3.841560738)+ (4.295543429)(4.28014389)+(3.680889086) (3.74917869) + (3.47475403) (3.809351106)

$$\sum_{i=1}^{4}\sqrt{\sum_{j=1}^{10}(\mathcal{H}_{ij}^{1})^2}\sqrt{\sum_{j=1}^{n}(\mathcal{H}_{ij}^{A})^2}$$

= 60.71416628

$$\therefore C_1=\sum_{i=1}^{4}\frac{\sum_{j=1}^{10}\mathcal{H}_{ij}^{1}\,\mathcal{H}_{ij}^{A}}{\sqrt{\sum_{j=1}^{10}(\mathcal{H}_{ij}^{1})^2}\sqrt{\sum_{j=1}^{10}(\mathcal{H}_{ij}^{A})^2}}$$

$$=\frac{60.58678701}{60.71416628}=0.997901984$$

$$\therefore C_1=0.997901984$$

Similarly for k=2, we get

$$C_2=\sum_{i=1}^{4}\frac{\sum_{j=1}^{10}\mathcal{H}_{ij}^{2}\,\mathcal{H}_{ij}^{A}}{\sqrt{\sum_{j=1}^{10}(\mathcal{H}_{ij}^{2})^2}\sqrt{\sum_{j=1}^{10}(\mathcal{H}_{ij}^{A})^2}}$$

$$\therefore C_2=\sum_{i=1}^{4}\frac{\sum_{j=1}^{10}\mathcal{H}_{ij}^{2}\,\mathcal{H}_{ij}^{A}}{\sqrt{\sum_{j=1}^{10}(\mathcal{H}_{ij}^{2})^2}\sqrt{\sum_{j=1}^{10}(\mathcal{H}_{ij}^{A})^2}}$$

$$=\frac{62.70206425}{62.82643551}=0.998020399$$

$$\therefore C_2=0.998020399$$

$$\sum_{k=1}^{2}C_k=C_1+C_2$$

$$=0.997901984+0.998020399$$

$$\sum_{k=1}^{2}C_k=1.995922383$$

∴ The decision maker weights are:

$$\gamma_1=\frac{C_1}{\sum_{k=1}^{t}C_k}=\frac{0.997901984}{1.995922383}=0.499970335$$

$$\gamma_1=0.5000$$

Similarly $\gamma_2=\frac{C_2}{\sum_{k=1}^{t}C_k}=\frac{0.998020399}{1.995922383}$

$$=0.500029664$$

$$\gamma_2=0.5000$$

$$\gamma_1+\gamma_2=0.5000+0.5000=1$$

Compute Collective hybrid score-accuracy matrix $\mathcal{H}$ by using decision maker's weight:

$\mathcal{H}_{ij} = \sum_{k=1}^{t} \gamma_k \mathcal{H}_{ij}^{k} \quad = \sum_{k=1}^{2} \gamma_k \mathcal{H}_{ij}^{k}$

The collective hybrid score accuracy matrix $\mathcal{H} = (\mathcal{H}_{ij})_{m\times n} = (\mathcal{H}_{ij})_{4\times 10}$ by using the hybrid score accuracy values of the two decision makers:

$\mathcal{H}$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
F_1	1.0833	1.3333	1.49995	1.24995	1.0667	1.2166	1.1083	1.2333	1.125	1.1667
F_2	1.5833	1.2917	1.5416	1.5	1.3583	1.3333	1.24995	1.2916	1.24995	1.05
F_3	1.3333	1.3333	1.025	1.0083	1.27495	1.3083	1.12495	1.2167	1.12495	1.0417
F_4	1.4167	1.4416	1.1917	1.1083	1.2916	1.1917	0.9833	1.1083	1.1083	1.12495

In this MCGDM problem find out the weights for the given criteria by using swing weighting technique. Calculate the weights for criteria $w_i = \frac{\delta_i}{\sum_j \delta_j}$

Criteria	Ordinal ranking	Formula	Weights
Simplicity in presentation	100	w_1 =100/550	0.18
Being on time	90	w_2 =90/550	0.16
Encouragement	70	w_3 = 70/550	0.13
Finishing the syllabus	65	w_4 = 65/550	0.12
Explanation of difficulties	50	w_5 = 50/550	0.09
Personalized care	45	w_6 = 45/550	0.08
Carrying out tests	40	w_7 = 40/550	0.07
Attention to students	35	w_8 = 35/550	0.06
Removing neutral	30	w_9 = 30/550	0.05
Application of ICT devices	25	w_{10} = 25/550	0.05
Sum	550 points		≅ 1

Compute weighted hybrid score-accuracy matrix $\mathcal{H}^W$:

$\mathcal{H}^W = (\mathcal{H}_{ij}^W)_{m\times n} \quad = (\mathcal{H}_{ij}^W)_{4\times 10}$

$\mathcal{H}_{ij}^W = w_j \mathcal{H}_{ij}$

The weighted hybrid score-accuracy matrix

$\mathcal{H}^W = (\mathcal{H}_{ij}^W)_{m\times n} \quad = (\mathcal{H}_{ij}^W)_{4\times 10}$

$\mathcal{H}^W$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
F_1	0.19499	0.2133	0.19499	0.14999	0.0960	0.0973	0.07758	0.07399	0.5625	0.05833
F_2	0.28499	0.20667	0.2004	0.18	0.12225	0.10667	0.0875	0.0775	0.0625	0.0525
F_3	0.23999	0.2133	0.13325	0.120996	0.1147	0.10467	0.07875	0.07300	0.05625	0.05209
F_4	0.25501	0.23066	0.154921	0.132996	0.11624	0.0953	0.06883	0.066498	0.05542	0.05625

Calculate the overall weighted hybrid score-accuracy values for each faculty $R(F_i) = \sum_{j=1}^{10} \mathcal{H}_{ij}^W$ where i=1,2,3,4.

$R(F_1) = \sum_{j=1}^{10} \mathcal{H}_{1j}^W$

$= \mathcal{H}_{11}^W + \mathcal{H}_{12}^W + \mathcal{H}_{13}^W + \mathcal{H}_{14}^W + \mathcal{H}_{15}^W + \mathcal{H}_{16}^W + \mathcal{H}_{17}^W + \mathcal{H}_{18}^W + \mathcal{H}_{19}^W + \mathcal{H}_{10}^W$

= 0.19499 + 0.2133 + 0.19499 +0.14999 + 0.0960 + 0.0973 + 0.07758 + 0.07399+0.05625 + 0.05833

$R(F_1)$ = 1.21272 ,$R(F_2)$ = 1.38098,$R(F_3)$ = 1.186996,$R(F_4)$ = 1.232125

According to the values of $R(F_1)$, $R(F_2)$, $R(F_3)$, $R(F_4)$ the ranking order of the faculties is $F_2 > F_4 > F_1 > F_3$. Finally, the decision makers decide F_2 is the outstanding faculty based on the students feedback system.

To conclude that, the students' feedback on faculty performance was analysed under indeterminacy in a unique way with neutrosophic environment. The outstanding faculty is found out with the help of decision makers by applying hybrid score accuracy functions of single-valued neutrosophic numbers and the method of ranking. It is concluded that according to the values of $R(F_1)$, $R(F_2)$, $R(F_3)$, $R(F_4)$ the ranking order of the faculty is $F_2 > F_4 > F_1 > F_3$.

The outstanding faculty based on the students' feedback system is F_2. In future, the outstanding faculty analysis using single valued neutrosophic environment may be done for large samples.

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