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MATE FUNCTIONS ON LEFT BIPOTENT AND S-NEAR RING

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ABSTRACT

In this paper we have discussed about left bipotent near rings and S-near rings with respect to mate functions. We have some results on left bipotent near rings using the concept of nilpotent, ideal, subdirect product and so on. It is proved that any homomorphic image of a left bipotent near ring is also such near ring. Also, it obtained that every left bipotent near ring has a mate function if and only if it is an S-near ring.

Keywords: Central, Idempotent, Left Bipotent, Mate function, Nilpotent, S-near ring.

INTRODUCTION

Near rings can be thought of as generalized rings: if in a ring we ignore the commutativity of addition and one distributive law, we get a near ring. "Near Rings" is an extensive collection of the work done in the area of near rings (Gratzer and George 1968 and Gunter Pilz, 1983). Suryanarayanan and Ganesan 1988 introduced the mate functions and the concept was then developed by Sugantha and Balakrishnan (2014) as β_1 Near-Rings. The concept of left bipotent and S-near ring was discussed by various mathematicians (Jat and Choudhary, 1979, Balakrishnan and Suryanarayanan 2000, Balakrishnan *et al* 2011 Radha *et al.*, 2019, Radha and Dhiva, 2019$_{a,b}$, 2020 and 2021). Using these concepts, we entrench some results.

Throughout this paper N stands for a right near ring $(N, +, .)$, with at least two elements and '0' denotes the identity element of the group $(N, +)$ and we write xy for x, y for any two elements x, y of N. Obviously $0n = 0$ for all $n \in N$. If, in addition, $n0 = 0$ for all $n \in N$ then we say that N is **zero symmetric.**

PRELIMINARIES

Definition 2.1 [4]

A **right near ring** is a non-empty set N together with two binary operations "+" and "." such that (i) $(N, +)$ is a group. (ii) $(N, .)$ is a semigroup (iii) $(n_1 + n_2)n_3 = n_1n_3 + n_2n_3$ for all $n_1, n_2, n_3 \in N$.

Definition 2.2 [5]

A near ring N is said to be **left bipotent** if $Na = Na^2$ for every$a$ in N.

Example 2.3 [5]

The near ring $(N, +, .)$ where $(N, +)$ is the group of integers modulo 4 and multiplication by the following table:

.	0	1	2	3
0	0	0	0	0
1	0	3	0	1
2	0	2	0	2
3	0	1	0	3

is a left bipotent near ring.

Definition 2.4 [1]

An element $a \in N$ is said to be **nilpotent**, if $a^k = 0$ for some positive integer k.

Definition 2.5 [2]

N is called an **S-near-ring (or) S'-near-ring** according as $x \in Nx$ (or) $x \in xN$ for all $x \in N$.

Example 2.6 [5]

Let $N = \{0,1,2,3,4,5,6\}$ with additions defined as addition modulo 7 and multiplication by the following table:

.	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	4	4	2	1
2	0	2	4	1	1	4	2
3	0	3	6	5	5	6	3
4	0	4	1	2	2	1	4
5	0	5	3	6	6	3	5
6	0	6	5	3	3	5	6

is a S-near ring.

Proposition 2.7 [5]

A left bipotent S-near ring has no non zero nilpotent elements.

Definition 2.8 [4]

An element $a \in N$ is said to be **idempotent** if $a^2 = a$.

Notation 2.9 [2]

1. E denotes the set of all idempotents of N.
2. L is the set of all nilpotent elements of N.
3. $C(N) = \{n \in N | nx = xn \text{ for all } x \in N\}$.

Definition 2.10 [4,13]

N is said to fulfil the **Insertion of Factors Property (IFP)** if $ab = 0 \Longrightarrow anb = 0$ for all a, b, n in N. If in addition $ab = 0 \Longrightarrow ba = 0$ for $a, b \in N$, we say N has $(*, \textbf{IFP})$.

Definition 2.11 [4]

Let N, N' be two near rings. Let $h: N \longrightarrow N'$ is called a **near-ring homomorphism** if for all $m, n \in N$

(i) $h(m + n) = h(m) + h(n)$

(ii) $h(mn) = h(m)h(n)$

Definition 2.12 [4]

A non-empty subset I of N is called

(i) a **left ideal** of N if $(I, +)$ is a normal subgroup of $(N, +)$ and $n(n' + i) - nn' \in N$ for all $n, n' \in N$ and $i \in I$.

(ii) a **right ideal** of N if $(I, +)$ is a normal subgroup of $(N, +)$ and if $IN \subseteq I$.

(iii) an **ideal** of N if I is both a left ideal and a right ideal of N.

Clearly $\{0\}$ and N are ideals of N. Theses are called **trivial ideals.**

Definition 2.13 [4]

A onto homomorphism is called an **epimorphism.**

Remark 2.14 [4]

If I is any ideal of N, then $\varphi : N \longrightarrow N/I$ defined by $\varphi(x) = I + x$ for $x \in N$ is a near ring epimorphism and is called **canonical homomorphism**.

Definition 2.15 [4]

Let $\{N_k | k \in K\}$ be a family of near rings. The Cartesian product of N_k with component wise defined operations '+' and '.' is called the **direct product** $\prod_{k\in K} \boldsymbol{N_k}$ of the near rings $N_k (k \in K)$.

Definition 2.16 [4]

(i) A sub direct product of near rings $\{N_k | k \in K\}$ is called **trivial** if there exists $k \in K$ such that the projection map π_k is an isomorphism.

(ii) N is called **sub directly irreducible** if N is not isomorphic to a non-trivial subdirect product of near rings.

Theorem 2.17 [4]

Each near ring N is isomorphic to a subdirect product of sundirectly irreducible near rings.

Definition 2.18 [2]

A map m from N into N is called a **mate function** for N, if $x = xm(x)x$ for all x in N. $m(x)$ is called a mate of x.

Example 2.19 [14]

Consider the near ring $(N, +, .)$ defined on the Klein's four group $(N, +)$ with $N = \{0, a, b, c\}$, where '.' is defined as follows (as per scheme 20, p. 408 of Pilz [4]).

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	a	b	c
c	a	0	c	b

This near ring admits mate function.

Main Results

Proposition 3.1

If N is a left bipotent near ring without nonzero nilpotent elements, then N has (*, IFP).

Proposition 3.2

Any homomorphic image of a left bipotent near ring is left bipotent near ring.

Proof:

Let $f: N \longrightarrow N'$ be a near ring epimorphism.

Let $x' \in N'$.

Then there exists $\in$ such that $f(x) = x'$.

Also, for $n' \in N'$there exists $n \in N$such that $f(n) = n'$.

Since N is left bipotent, $Nx = Nx^2$...... (1).

Now

$n'x' = f(n)\, f(x)$

$= f(nx)$ [f is a homomorphism]

$= f(n_1x^2)$ [Since by equation (1) for some n_1 in N]

$= f(n_1)\, f(x^2)$ [Since$f$ is homomorphism]

$= f(n_1)\, [f(x)]^2$

$= n_1 x'^2$

(i.e)$n'x' = n_1' x'^2$

Therefore $n'x' \in N'x'^2$.

Consequently $N'x' \subseteq N'x'^2$ ...... (2)

Similarly, $N'x'^2 \subseteq N'x'$ .... (3)

Combining equations (2) & (3) we get

$N'x' = N'x'^2$

Hence N' is also a left bipotent near ring and the desired result follows.

Corollary 3.3

Let N be a left bipotent near ring.If I is any ideal of N, then N/I is also a left bipotent near ring.

Proof:

Let $f: N \longrightarrow N/I$ be the canonical near ring homomorphism given by $f(x) = I + x$. Clearly f is onto. The rest of the proof is taken care of by Proposition 3.2.

Proposition 3.4

Every left bipotent near ring N is isomorphic to a sub direct product of sub directly irreducible left bipotent near rings.

Proof:

By Theorem 2.17, N is isomorphic to a subdirect product of subdirectly irreducible near rings N_i's and each N_i is a homomorphic image of N under the projection mapπ_i.Now the desired result follows from proposition 3.2.

Proposition 3.5

Let N be a left bipotent near ring.Then N has a mate function if and only if N is an S-near ring.

Proof:

When N has a mate function 'm' for all $x \in N$, $x = xm(x)x \in Nx$.

This implies $x \in Nx$.

Hence N is an S-near ring.

Conversely, let N be an S-near ring,

Then $x \in Nx = Nx^2$ for all $x \in N$.

$\Longrightarrow x \in Nx^2$

$\Longrightarrow x = nx^2$ for some $n \in N$.

$\Longrightarrow x^2 = xnx^2$

$\Longrightarrow x^2 - xnx^2 = 0$

$\Longrightarrow (x - xnx)x = 0$.

$x(x - xnx) = 0$ & $xnx\,(x - xnx) = 0$

(By Proposition 2.7 & Proposition 3.1)

Now $(x - xnx)^{\,2} = (x - xnx)\,(x - xnx)$

$= x(x - xnx) - xnx\,(x - xnx)$

$= 0$

(i.e) $(x - xnx)^2 = 0$.

Since $L = \{0\}$ implies $x - xnx = 0$.

(i.e) $x = xnx$

This implies $x = xm(x)x$ where $n = m(x)$.

Hence $m: N \longrightarrow N$ is a mate function for N.

By Proposition 2.7, Proposition 3.5 & Proposition 3.1 we get the following proposition.

Proposition 3.6

If N hasa left bipotent near ring and a mate function 'm' then $L = \{0\}$ and N has (*, IFP).

Proposition 3.7

Let N be a left bipotent near ring. If N is subcommutative then $E \subseteq C(N)$.

Proof:

Let N be a left bipotent near ring.

Then $Na = Na^2$ for all $a \in N$.

For $e \in E, Ne = Ne^2$.

$\Longrightarrow eN = Ne = Ne^2$(Since N is subcommutative)

$$\Longrightarrow eN = Ne$$

$\Longrightarrow en = ne$ for all $n \in N$.

(i.e) $E \subseteq C(N)$.

Theorem 3.8

Let N be a near ring with a mate function 'm'. If $E \subseteq C(N)$ then N is left bipotent.

Proof:

Let $a \in N$.

Then $na \in Na$ for all $n \in N$.

Since $E \subseteq C(N)$

$na = n(am(a)a)$

$= na(m(a)a)$

$= na(m(a)am(a)a)$ (Since $m(a)a \in E$)

$= nam(a)m(a)aa$

$= na[m(a)]^2a^2$

$\in Na^2$

Therefore $Na \subseteq Na^2$...... (1)

Also $na^2 \in Na^2$.

$\Longrightarrow na^2 = naa$

$= n(am(a)a)a$

$= nam(a)a^2$

$\in Na$

Therefore $Na^2 \subseteq Na$ ...... (2)

From equation (1) &equation (2) we get,

$Na = Na^2$ for all $a \in N$.

Hence N is left bipotent.

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