International Multidisciplinary Innovative Research Journal
-An International refereed e-journal

ISSN: 2456 – 4613
Volume – VI Issue - 2
May - 2022

CLOSED SUPPORT STRONG EFFICIENT DOMINATION NUMBER OF SOME STANDARD GRAPHS UNDER ADDITION AND MULTIPLICATION

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ABSTRACT

Let G = (V, E) be a graph with p points and q nodes. Let S be a γ_{se}- set of G. Let v ϵ S. A closed support strong efficient domination number of v under addition is defined by $\sum_{u \epsilon N[v]} \deg(u)$ and it is denoted by supp $\gamma_{se}{}^{+}[v]$. A closed support strong efficient domination number of G under addition is defined by $\sum_{v \epsilon S} \text{supp}\ \gamma_{se}{}^{+}[v]$and it is denoted by supp $\gamma_{se}{}^{+}[G]$. A closed support strong efficient domination number of v under multiplication is defined by $\prod_{u \epsilon N[v]} \deg(u)$ and it is denoted by supp $\gamma_{se}{}^{\times}[v]$. A closed support strong efficient domination number of G under multiplication is defined by $\prod_{v \epsilon S} \text{supp}\ \gamma_{se}{}^{\times}[v]$and it is denoted by supp $\gamma_{se}{}^{\times}[G]$. In this paper, closed support strong efficient domination number of some standard graphs is studied.

Keywords*:* domination, addition, graph, multiplication, theorem

INTRODUCTION

Throughout this paper only finite, undirected graphs without loops or multiple nodes are considered. Let G = (V, E) be a graph with p points and q nodes. The degree of any point v of a graph G is the number of nodes incident with v and is denoted by deg (v). A subset S of V(G) is called a dominating set of G if every point in V(G) $-$ S is adjacent to a point in S (see [6]). The domination number of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G. Sampathkumar et.al. introduced the concept of strong (weak) domination in graphs (see [11]). A subset S of V(G) is called a strong dominating set of G if for every v $\in V(G) - S$ there exists a point u $\in S$ such that u and v are adjacent and deg(u) ≥ deg(v). A subset S of V(G) is called an efficient dominating set if for every v $\in V(G), |N[v] \cap S| = 1$ (see [3, 5]). The concept of strong (weak) efficient domination in graphs was introduced by Meena et.al. (see[10]) and further studied in(see [7, 8, 9]). A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every point $v \in V(G)$, we have $|N_s[v] \cap S| = 1$ ($|N_W[v] \cap S| = 1$), where $N_s(v) = \{u \in V(G); uv \in E(G), \deg(u) \geq \deg(v)\}$ and $N_s[v] = N_s(v) \cup \{v\}$($N_w(v) = \{u \in V(G); uv \in E(G), \deg(u) \leq \deg(v)\}$ and $N_w[v] = N_w(v) \cup \{v\}$). The minimum cardinality of a strong (weak) efficient dominating set of G is called the strong (weak) efficient domination number of G and is denoted by $\gamma_{se}(G)(\gamma_{we}(G))$. A graph G is strong efficient if there exists a strong efficient dominating set of G. Balamurugan et.al introduced the concept of closed support of a graph under addition (see [1]) and multiplication (see [2]). A closed support of a point v under addition is defined by $\sum_{u \in N[v]} \deg(u)$ and it is denoted by $supp[v]$. A closed support of a graph G under addition is defined by $\sum_{v \in V(G)} supp[v]$ and it is denoted by $supp[G]$. A closed support of a point v under multiplication is defined by $\prod_{u \in N[v]} \deg(u)$ and is denoted by $mult[v]$. A closed support of a graph G under multiplication is defined by $\prod_{u \in V(G)} mult[v]$ and it is denoted by $mult[G]$. Inspired by the above definitions, the concept of a closed support strong efficient domination number of a graph under addition and multiplication are introduced in this paper. For all Graph theoretic terminologies and notations, Harary (see [4]) is followed. Following previous results are necessary for the present study.

Previous results [9]

1. For any path P_m ,
$$\gamma_{se}(P_m)\begin{cases} n \text{ if } m = 3n, n \in N \\ n+1 \text{ if } m = 3n+1, n \in N \\ n+2 \text{ if } m = 3n+2, n \in N \end{cases}$$
2. For any cycle C_{3n}, $\gamma_{se}(C_{3n})$ = n, n ∈ N
3. $\gamma_{se}(K_{1,n})$ = 1, n ∈ N
4. $\gamma_{se}(K_n)$ = 1, n ∈ N
5. $\gamma_{se}(D_{m,n})$ = m + 1, m ≤ n, m, n ∈ N

MAIN RESULTS

Definition 2.1: Let G = (V, E) be a strong efficient graph. Let S be a γ_{se}- set of G. Let v ϵ S. A closed support strong efficient domination number of v under addition is defined by $\sum_{u\epsilon N[v]} \deg(u)$ and it is denoted by $supp\gamma_{se}{}^{+}[v]$.

Example 2.2: Consider the following graph G.

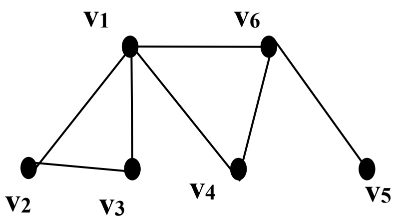

Fig 1

S = $\{v_1, v_5\}$ is the unique γ_{se} - set of G.

$supp \quad \gamma_{se}{}^{+}[v_1] = \sum_{u\epsilon N[v_1]} \deg(u)$ = $\deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4)+$ $\deg(v_6)$= 13.

$suppp \quad \gamma_{se}{}^{+}[v_5] = \sum_{u\epsilon N[v_5]} \deg(u \quad)$ = $\deg(v_5)$+ $\deg(v_6)$ = 4

Definition 2.3: Let G = (V,E) be a strong efficient graph. Let S be a γ_{se}- set of G. Let v ϵ S. A closed support strong efficient domination number of v under multiplication is defined by $\prod_{u\epsilon N[v]} \deg(u)$ and it is denoted by $supp\gamma_{se}{}^{\times}[v]$.

Example 2.4: In Fig. 1, $supp\,\gamma_{se}{}^{\times}[v_1]$ = $\prod_{u\epsilon N[v_1]} \deg(u)$ = $\deg\ (v_1) \times \deg\ (v_2) \times \deg\ v_3) \times$ $\deg(v_4)$ ×$\deg(v_6)$= 96

$supp \quad \gamma_{se}{}^{\times}[v_5] = \prod_{u\epsilon N[v_5]} \deg(u)$ = $\deg(v_5) \times \deg(v_6)$ = 3

Definition 2.5: Let G be a strong efficient graph. Let S be a γ_{se}- set of G. A closed support strong efficient domination number of G under addition is defined by $\sum_{v\epsilon S} \text{supp}\,\gamma_{se}{}^{+}[v]$ and it is denoted by $supp\gamma_{se}{}^{+}[G]$

Example 2.6: In Fig. 1, $supp\,\gamma_{se}{}^{+}[G]$ = $supp\gamma_{se}{}^{+}[v_1]$ + $supp\gamma_{se}{}^{+}[v_5]$ = 17

Definition 2.7: Let G be a strong efficient graph. Let S be a γ_{se}- set of G. A closed support strong efficient domination number of G under multiplication is defined by $\prod_{v\epsilon S} \text{supp}\,\gamma_{se}{}^{\times}[v]$ and it is denoted by $supp\gamma_{se}{}^{\times}[G]$.

Example 2.8: In Fig. 1, $supp\,\gamma_{se}{}^{\times}[G]$ = $supp\gamma_{se}{}^{\times}[v_1]$ ×$supp\gamma_{se}{}^{\times}[v_5]$ = 288.

Note 2.9: Closed support strong efficient domination number under addition of a graph G is not unique.

Example 2.10: Consider the following graph G

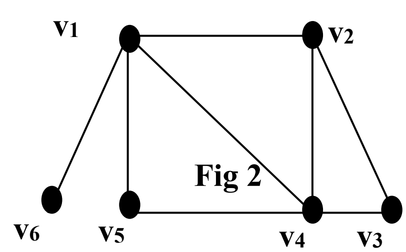

Fig 2

$S_1 = \{v_1, v_3\}$ and $S_2 = \{v_4, v_6\}$ are two γ_{se} - sets of G and γ_{se} (G) = 2. For S_1 , $supp\ \ {\gamma_{se}}^{+}[v_1] = \sum_{u\in N[v_1]} \deg(u) = \deg(v_1) + \deg(v_2) + \deg(v_4) + \deg(v_5) + \deg(v_6) = 13$.

$supp\ \ {\gamma_{se}}^{+}[v_3] = \sum_{u\in N[v_3]} \deg(u) = \deg(v_3) + \deg(v_2) + \deg(v_4) = 9$

$supp\ \ {\gamma_{se}}^{+}[G] = \sum_{v\epsilon S_1} \text{supp}\, {\gamma_{se}}^{+}[v] = supp{\gamma_{se}}^{+}[v_1] + supp{\gamma_{se}}^{+}[v_3] = 22$

For S_2 , $supp\, {\gamma_{se}}^{+}[v_4] = \sum_{u\in N[v_4]} \deg(u) = \deg(v_4) + \deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_5) = 15$

$supp\ \ {\gamma_{se}}^{+}[v_6] = \sum_{u\in N[v_6]} \deg(u) = \deg(v_6) + \deg(v_1) = 5$

$supp\ \ {\gamma_{se}}^{+}[G] = \sum_{v\epsilon S_2} \text{supp}\, {\gamma_{se}}^{+}[v] = supp{\gamma_{se}}^{+}[v_4] + supp{\gamma_{se}}^{+}[v_6] = 20$.

Hence min $supp\, {\gamma_{se}}^{+}[G] = 20$ and max $supp{\gamma_{se}}^{+}[G] = 23$.

Note 2.11: Closed support strong efficient domination number under multiplication of a graph G is not unique.

Example 2.12: Consider Fig.2,

$S_1 = \{v_1, v_3\}$ and $S_2 = \{v_4, v_6\}$are two γ_{se} - sets of G and $\gamma_{se}(G) = 2$

For S_1, $supp\, {\gamma_{se}}^{\times}[v_1] = \prod_{u\in N[v_1]} \deg(u) = \deg(v_1) \times \deg(v_2) \times \deg(v_4) \times \deg(v_5) \times \deg(v_6) = 72$

$supp\ \ {\gamma_{se}}^{\times}[v_3] = \prod_{u\in N[v_3]} \deg(u) = \deg(v_3) \times \deg(v_2) \times \deg(v_4) = 24$

$supp\, {\gamma_{se}}^{\times}[G] = \prod_{v\epsilon S_1} \text{supp}\, {\gamma_{se}}^{\times}[v]$ = supp ${\gamma_{se}}^{\times}[v_1]$ ×supp ${\gamma_{se}}^{\times}[v_3]$ =1728.

For S_2 , $supp\, {\gamma_{se}}^{\times}[v_4] = \prod_{u\in N[v_4]} \deg(u) = \deg(v_4) \times \deg(v_1) \times \deg(v_2) \times \deg(v_3) \times \deg(v_5) = 192$

$supp\ \ {\gamma_{se}}^{\times}[v_6] = \prod_{u\in N[v_6]} \deg(u) = \deg(v_6) \times \deg(v_1) = 4$

$supp\ \ {\gamma_{se}}^{\times}[G] = \prod_{v\epsilon S_2} \text{supp}\, {\gamma_{se}}^{\times}[v] = supp{\gamma_{se}}^{\times}[v_4] \times supp{\gamma_{se}}^{\times}[v_6] = 768$.

Here min $supp\, {\gamma_{se}}^{\times}[G] = 768$ and max $supp{\gamma_{se}}^{\times}[G] = 1728$.

Remark 2.13: Let G be a connected strong efficient graph with aγ_{se}- set S. Since S ⊂ V(G), $supp\ \ {\gamma_{se}}^{+}[G] < supp[G]$ and $supp{\gamma_{se}}^{\times}[G] < mult[G]$

Theorem 2.14: Let G = $P_{3n,}\ n \in N$. Then

i. $supp{\gamma_{se}}^{+}[G]$ = 6n – 2 and

ii. $supp{\gamma_{se}}^{\times}[G] = 2(8^{n-1})$

Proof: Let G = $P_{3n,}\ n \in N$. Let V(G) = $\{v_i;\ 1 \leq i \leq 3n\}$. Then S =$\{v_2, v_5, v_{8,,} \dots, v_{3n-4}, v_{3n-1}\}$ is the unique γ_{se}- set of G. $\deg(v_1) = \deg(v_{3n}) = 1$ and $\deg(v_i) = 2$, 2 ≤ i ≤ 3n – 1.

i. $supp\ \ {\gamma_{se}}^{+}[v_2] = \deg(v_2) + \deg(v_1) + \deg(v_3) = 5$

For i = 5, 8,…,3n – 4, supp ${\gamma_{se}}^{+}[v_i] = \deg(v_i) + \deg(v_{i-1}) + \deg(v_{i+1}) = 6$

$supp\ \ {\gamma_{se}}^{+}[v_{3n-1}] = \deg\ (v_{3n-1}) + \deg(v_{3n-2}) + \deg(v_{3n}) = 5$

Hence $supp\ \gamma_{se}{}^{+}[G] = supp\ \gamma_{se}{}^{+}[v_2] + \sum_{i=1}^{n-2} \text{supp}\ \gamma_{se}{}^{+}[v_{3i+2}] + supp\ \gamma_{se}{}^{+}[v_{3n-1}]$ = 5 + (n – 2)6 + 5 = 6n – 2.

ii. $supp\ \gamma_{se}{}^{\times}[v_2] = \deg(v_2) \times \deg(v_1) \times \deg(v_3) = 4$

For i = 5,8,…,3n – 4, $supp\ \gamma_{se}{}^{\times}[v_i] = \deg(v_i) \times \deg(v_{i-1}) \times \deg(v_{i+1}) = 8$

$supp\ \gamma_{se}{}^{\times}[v_{3n-1}] = \deg(v_{3n-1}) \times \deg(v_{3n-2}) \times \deg(v_{3n}) = 4$.

Hence $supp\ \gamma_{se}{}^{\times}[G] = supp\ \gamma_{se}{}^{\times}[v_2] \times \prod_{i=1}^{n-2}. \text{supp}\ \gamma_{se}{}^{\times}[v_{3i+2}] \times supp\ \gamma_{se}{}^{\times}[v_{3n-1}]$ = $4 \times 8^{n-2} \times 4 = 2(8^{n-1})$.

Theorem 2.15: Let G = $P_{3n+1}, n \in N$. Then

i. $supp\gamma_{se}{}^{+}[G]$ = 6n + 2 and

ii. $supp\gamma_{se}{}^{\times}[G] = 8^{n}$

Proof: Let G = $P_{3n+1}, n \in N$. Let V(G) = $\{v_i;\ 1 \le i \le 3n+1\}$.

$S_1 = \{ v_1, v_3, v_{6,} \dots, v_{3n-3}, v_{3n} \}$ and $S_2 = \{v_2, v_5, v_{8,} \dots, v_{3n-1}, v_{3n+1}\}$ are two distinct γ_{se}- sets of G.$\deg(v_1) = \deg(v_{3n+1}) = 1$ and $\deg(v_i) = 2$, 2 ≤ i ≤ 3n.

Consider S_1. (Proof is similar for S_2)

i. $supp\ \gamma_{se}{}^{+}[v_1] = \deg(v_1) + \deg(v_2) = 3$

For i = 3,6,…,3n – 3, $supp\ \gamma_{se}{}^{+}[v_i] = \deg(v_i) + \deg(v_{i-1}) + \deg(v_{i+1}) = 6$

$supp\gamma_{se}{}^{+}[v_{3n}] = \deg(v_{3n}) + \deg(v_{3n-1}) + \deg(v_{3n+1}) = 5$

Hence $supp\ \gamma_{se}{}^{+}[G] = supp\ \gamma_{se}{}^{+}[v_1] + \sum_{i=1}^{n-1} \text{supp}\ \gamma_{se}{}^{+}[v_{3i}] + supp\gamma_{se}{}^{+}[v_{3n}] = 3$ + (n – 1)6 + 5 = 6n + 2.

ii. $supp\gamma_{se}{}^{\times}[v_1] = \deg(v_1) \times \deg(v_2) = 2$

For i = 3,6,…,3n – 3, $supp\ \gamma_{se}{}^{\times}[v_i] = \deg(v_i) \times \deg(v_{i-1}) \times \deg(v_{i+1}) = 8$

$supp\ \gamma_{se}{}^{\times}[v_{3n}] = \deg(v_{3n}) \times \deg(v_{3n-1}) \times \deg(v_{3n+1}) = 4$

$supp\ \gamma_{se}{}^{\times}[G] = supp\ \gamma_{se}{}^{\times}[v_1] \times \prod_{i=1}^{n-1} \text{supp}\ \gamma_{se}{}^{\times}[v_{3i}] \times supp\ \gamma_{se}{}^{\times}[v_{3n}]$ = $2\times 8^{n-1} \times 4 = 8^{n}$.

Hence $supp\ \gamma_{se}{}^{+}[G]$ = 6n + 2 and $supp\gamma_{se}{}^{\times}[G] = 8^{n}$.

Theorem 2.16: Let G = $P_{3n+2}, n \in N$. Then

i. $supp\gamma_{se}{}^{+}[G]$ = 6(n + 1) and

ii. $supp\gamma_{se}{}^{\times}[G] = 4(8^{n})$

Proof: Let G = $P_{3n+2}, n \in N$. Let V(G) = $\{v_i;\ 1 \le i \le 3n+2\}$. Then S $= \{ v_1, v_3, v_6, \dots, v_{3n}, v_{3n+2} \}$ is the unique γ_{se}- set of G. $\deg(v_1) = \deg(v_{3n+2}) = 1$ and $\deg(v_i) = 2$, 2 ≤ i ≤ 3n + 1.

i. $supp\gamma_{se}{}^{+}[v_1] = \deg(v_1) + \deg(v_2) = 3$. For i = 3, 6,…, 3n, $supp\ \gamma_{se}{}^{+}[v_i] = \deg(v_i) + \deg(v_{i-1}) + \deg(v_{i+1}) = 6$

$supp\ \gamma_{se}{}^{+}[v_{3n+2}] = \deg(v_{3n+2}) + \deg(v_{3n+1}) = 3$

Hence $supp\ \gamma_{se}{}^{+}[G] = supp\ \gamma_{se}{}^{+}[v_1] + \sum_{i=1}^{n} \text{supp}\ \gamma_{se}{}^{+}[v_{3i}] + supp\gamma_{se}{}^{+}[v_{3n+2}] = 3$ + 6n + 3 = 6(n + 1).

ii. $supp\gamma_{se}^{\times}[v_1] = \deg(v_1) \times \deg(v_2) = 2$. For i = 3,6,…,3n, $supp\,\gamma_{se}^{\times}[v_i] = \deg(v_i) \times \deg(v_{i-1}) \times \deg(v_{i+1}) = 8$.

$supp\ \gamma_{se}^{\times}[v_{3n+2}] = \deg\ (v_{3n+2}) \times \deg(v_{3n+1}) = 2$

Hence $supp\ \gamma_{se}^{\times}[G] = supp\ \gamma_{se}^{\times}[v_1] \times \prod_{i=1}^{n} \text{supp}\ \gamma_{se}^{\times}[v_{3i}] \times supp\ \gamma_{se}^{\times}[v_{3n+2}] = 2 \times 8^n \times 2 = 4(8^n)$.Hence $supp\gamma_{se}^{+}[G] = 6n + 2$ and $supp\gamma_{se}^{\times}[G] = 4(8^n)$.

Theorem 2.17: Let G = $C_{3n}, n \in N$. Then

i. $supp\gamma_{se}^{+}[G] = 6n$ and

ii. $supp\gamma_{se}^{+}[G] = 8^n$

Proof: Let G = $C_{3n}, n \in N$. Let V(G) = $\{v_i;\ 1 \le i \le 3n\}$. Then $S_1 = \{v_1, v_4, v_7, \ldots, v_{3n-2}\}$, $S_2 = \{v_2, v_5, v_8, \ldots, v_{3n-1}\}$ *and* $S_3 = \{v_3, v_6, v_9, \ldots, v_{3n}\}$ are three distinct γ_{se} -sets of G. $\deg(v_i) = 2$, 1 ≤ i ≤ 3n.Consider $S_1 = \{v_1, v_4, v_7, \ldots, v_{3n-2}\}$. (Proof is similar for$S_2$ and S_3)

i. For i = 1, 4, 7,…3n – 2, $supp\ \gamma_{se}^{+}[v_i] = \deg\ (v_i) + \deg\ (v_{i-1}) + \deg(v_{i+1}) = 6$

Hence $supp\ \gamma_{se}^{+}[G] = \sum_{i=1}^{n} \text{supp}\ \gamma_{se}^{+}[v_{3i-2}] = 6n$

ii. For i = 1, 4, 7,…3n – 2, $supp\ \gamma_{se}^{\times}[v_i] = \deg\ (v_i) \times \deg\ (v_{i-1}) \times \deg(v_{i+1}) = 8$

Hence $supp\ \gamma_{se}^{\times}[G] = \prod_{i=1}^{n} \text{supp}\ \gamma_{se}^{\times}[v_{3i-2}] = 8^n$.

Hence $supp\gamma_{se}^{+}[G] = 6n$ and $supp\gamma_{se}^{\times}[G] = 8^n$.

Theorem 2.18: Let G = $K_{1,n}, n \in N$. Then

i. $supp\gamma_{se}^{+}[G] = 2n$ and

ii. $supp\gamma_{se}^{\times}[G] = n$

Proof: Let G = $K_{1,n}, n \in N$. Let V(G) =$\{v, v_i;\ 1 \le i \le n\}$ where v is the central point. Then S = {v}is the unique γ_{se}- set of G. $\deg(v)$ = n and $\deg(v_i) = 1$, $1 \le i \le n$

i. $supp\ \gamma_{se}^{+}[v] = \deg\ (v) + \sum_{i=1}^{n} \deg[v_i]$ = n + n(1) = 2n

Hence supp $\gamma_{se}^{+}[G]$ = supp $\gamma_{se}^{+}[v]$ = 2n

ii. $supp\ \gamma_{se}^{\times}[v] = \deg\ v \times \prod_{i=1}^{n} \deg(v_i) = n \times 1^n = n$

Hence $supp\gamma_{se}^{\times}[G] = supp\gamma_{se}^{\times}[v] = n$

Theorem 2.19: Let G = $K_n,\ n \in N$. Then

i. $supp\gamma_{se}^{+}[G] = n(n-1)$and

ii. $supp\gamma_{se}^{\times}[G] = (n-1)^n$

Proof: Let G = $K_n, n \in N$. Let v_i; $1 \le i \le n$ be the points of G. Then $S_i = \{v_i\}$, $1 \le i \le n$ are n distinct γ_{se}- setsof G. $\deg(v_i)$ = n – 1, $1 \le i \le n$.

Consider the set S_i (Proof is similar for the other sets)

i. $supp\gamma_{se}^{+}[v_i] = \sum_{i=1}^{n} \deg(v_i) = n(n-1)$. Hence supp $\gamma_{se}^{+}[G]$ = supp $\gamma_{se}^{+}[v_i = n(n-1)$

ii. $supp\ \gamma_{se}^{\times}[v_i] = \prod_{i=1}^{n} \deg(v_i) = (n-1)^n$. Hence $supp\ \gamma_{se}^{\times}[G] = supp\gamma_{se}^{\times}[v_i] = (n-1)^n$

Hence $supp\ \gamma_{se}{}^{+}[G] = n(n-1)$and $supp\gamma_{se}{}^{\times}[G] = (n-1)^n$

Remark 2.20: Let G be a nontrivial connected strong efficient graph on n ≥ 2 points. Then $1 \leq supp\gamma_{se}{}^{+}[G] \leq n(n-1)$and $1 \leq supp\gamma_{se}{}^{\times}[G] \leq (n-1)^n$.

Theorem 2.21: Let G = $D_{m,n}$,m ≥ n and m, n ∈ N. Then

i. $supp\ \gamma_{se}{}^{+}[G] = (n+1)^2 + 2m + n + 1$and

ii. $supp\gamma_{se}{}^{\times}[G] = (m+1)(n+1)^{n+1}$

Proof: Let G = $D_{m,n}$, m ≥ n. Let V(G) =$\{u, u_i, v, v_j;\ 1 \leq i \leq m,\ 1 \leq j \leq n\}$ Let E(G) = {uv, uu_i, vv_j; 1≤ $i \leq m$, 1≤ $i \leq n\}$. Then $S = \{u, v_j; 1 \leq j \leq n\}$is the unique γ_{se}- set of G. deg (u) = m + 1, deg (v) = n + 1, deg (u_i) = deg (v_j) = 1, $1 \leq i \leq m$, $1 \leq j \leq n$.

$supp\,\gamma_{se}{}^{+}[u]$= deg (u) + $\sum_{i=1}^{m} \deg(u_i)$ + deg (v) = m + 1 + m + n + 1 = 2m + n + 2

For j = 1,2,…,n, $supp\,\gamma_{se}{}^{+}(v_j)$ =$\deg(v_j)$ + deg (v) = n + 2

$supp\ \gamma_{se}{}^{+}[G] = supp\ \gamma_{se}{}^{+}[u] + \sum_{j=1}^{n} \text{supp}\ \gamma_{se}{}^{+}(v_j)$ = 2m+ n+2+n(n + 2) = $(n+1)^2 + 2m + n + 1$

$supp\ \gamma_{se}{}^{\times}[u]$ = deg $(u) \times \prod_{i=1}^{m} \deg(u_i) \times$ deg (v) = (m + 1) (n + 1)

For j = 1,2,…,n, $supp\ \gamma_{se}{}^{\times}[v_j]$ = $\deg(v_j)$ ×$\deg(v)$ = n + 1

$supp\ \gamma_{se}{}^{\times}[G] = supp\ \gamma_{se}{}^{\times}[u] \times \prod_{j=1}^{n} \text{supp}\ \gamma_{se}{}^{\times}[v_j]$ = (m + 1)(n + 1)$(\mathrm{n}+1)^n$ = $(m+1)(n+1)^{n+1}$

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