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THE ROLE OF WEAK COMMUTATIVITY IN BOOLEAN LIKE NEAR-RING

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ABSTRACT

The concepts of Boolean like near-ring and Special Boolean like near-ring are introduced by Clay, James.R and Lawver, Donald, A., during 1969. In this paper, we have discussed the concept of weak commutative property in a Boolean like near-ring. If R is a weak commutative near ring, $(ab)^n = a^n b^n$ for all a,b in R and for $n \leq 1$ and $(a-a^2)(b-b^2)c = 0$ for every a,b,c in a Boolean like near-ring. Also, it is proved that, every near-ring is reduced if it has Weak Commutativity and also satisfies (*, IFP) property. If R be a weak commutative Boolean Like near-ring and S be a commutative subset with multiplicatively closed. Then we define a relation N on R×S by $(r_1, s_1) \sim (r_2, s_1)$ if there exists an element $s \in S$ such that $s(r_1 s_2 - r_2 s_1) = 0$. Then N is an equivalence relation. And also define binary operation + an on $S^{-1}R$ as,

$\frac{r_1}{s_2} + \frac{r_2}{s_2} = \frac{r_1 s_2 + r_2 s_1}{s_1 s_2}$ and $\frac{r1}{s_2} \cdot \frac{r_2}{s_2} = \frac{r_1 r_2}{s_1 s_2}$ Then $S^{-1}R$ is a commutative Boolean Like near-ring with identity and also a Weak Commutative near-ring.

Keywords: Near-Rings, Weak Commutative, Reduced, Boolean Like Near-Ring, *R*-Subgroup, IFP, (*,IFP), strong IFP.

PRELIMINARIES

Definition 2.1

A non-empty set R with two binary operations "$+$" (addition) and "$\cdot$" (multiplication) is satisfying the following axioms is called a *right near-ring*

(i) $(R, +)$ is a $(R, \cdot)$ is a semi group.
(ii) For all x,y,z, in R, $(x + y).z = x.z + y.z$(right distributive law)
(iii) group (not necessarily abelian).

Definition 2.2

A near ring R is called a *zero-symmetric* if $ab = 0$ implies $ba = 0$,where $a, b \epsilon R$.

Note 2.3

In a right near-ring R, $0. a = 0 \; \forall a \in R$.

If $(R, +)$ is an abelian group, then $(R, +,\cdot)$ is called a semi-ring.

Definition 2.4

A near-ring R is subset H if R such that

(i) $(H, +)$ is a subgroup of $(R, +)$

(ii) $RH \subseteq H$

(iii) $HR \subseteq H$

If H satisfies (i) and (ii) then it is called left *R-subgroup of R*. If H satisfies (i) and(iii) then H is called a right *R*-subgroup of *R*.

Definition 2.5

A near-ring *R* is said to *reduced* if *R* has no non-zero nilpotent elements.

Definition 2.6

Let *R* be a near-ring. *R* is said to satisfy *intersection of factors property* (IFP) if $ab = 0 \Rightarrow anb = 0$ for all $n \in R$, where $a, b \in R$.

Definition 2.7

R is said to have strong IFP, if for all ideals I of R, $ab \in I \quad \Rightarrow \quad anb \in I \; for \; all \;\; n \in R$

Definition 2.8

A near-ring *R* is said to be *regular* near-ring if for every *a* in R there exists x in R such that $a = axa$.

Definition 2.9

A right near ring *R* is said to be *Weak commutative* if $xyz = xzy \; \forall x, y, z \in R$

Definition 2.10

A right near-ring $(R, +,\cdot)$ is called a *Boolean-like near ring* if

(i) $2a = 0 \; \forall \; a \in R$ and

(ii) $(a + b - ab) = ab \; \forall \; a, b \in R$

THE ROLE OF WEAK COMMUTATIVITY IN BOOLEAN LIKE NEAR-RING

Lemma 3.1

Let *R* be a weak commutative near-ring *R*. Then $(ab) = a^n b^n \; \forall \; a, b \; \in R$ and $\forall \; n \geq 1$

Proof:

Let $a, b \in R$

Then $(ab)^2 = (ab)(ab) = (aba)b$

$= (aab)$ (R is weak commutative)

$= a^2b^2$

Assume that $(ab) = a^m b^m$

No $(ab)^{+1} = (ab)^m ab$

$= a^m b^m\, ab = (\, a^m b^m\, a)b$

$= (a^m\,)$ (R is weak commutative)

$= a^{m+1} b^{m+1}$

Thus $(ab) = a^m b^m \,\forall\, a, b \in R$ and for all integer $m \geq 1$

Lemma 3.2

Let R be a weak commutative Boolean like near-ring. Then $a^2b + ab^2 = ab + (ab)^2 \forall\, a, b \in R$.

Proof:

$a^2b + ab^2 = aab + abb$

$= aba + abb$

$= ab(a + b)$

$= ab(a + b - ab + ab)$

$= (a + b - ab) + (ab)^2$

$= ab + (ab)^2$ (R is Boolean like near-ring)

$a^2b + ab^2 = ab + (ab)^2 \forall\, a, b \in R$.

Lemma 3.3

In a weak commutative Boolean like near-ring $(R, +, \cdot\,)$

Then $(a + a^2)(b + b^2)c = 0 \;\forall\, a, b, c \in R$.

Proof: $(a + a^2)(b + b^2)c = \{\, a\,(b + b^2) + a^2(b + b^2)\}c$

$= a\,(b + b^2) + a^2(b + b^2)c$

$= ac(b + b^2) + a^2(b + b^2)\,\}$ (R is weak commutative)

$= \mathrm{c}\{\, a\,(b + b^2) + a^2(b + b^2)\}$

$= c\{ ab + ab^2 + a^2b + a^2 b^2\}$
$= c\{ab + ab + (ab)^2 + a^2b^2\}$ (using Lemma 3.2)
$= c\{2ab + 2a^2b^2\}$
$= 0$ (R is Boolean like near-ring)

Lemma 3.4

In a weak commutative Boolean like near-ring R,

$(a - a^2)(b - b^2)c = 0 \ \forall\ a, b, c \in R$

Proof: $(a - a^2)(b - b^2)c = \{(b - b^2) - a^2(b - b^2)\}c$

$= \{(b - b^2) - a^2(b - b^2)c\}$

$= \{(b - b^2) - a^2(b - b^2)\}$ (R is weak commutative)

$= \{(b - b^2) - a^2(b - b^2)\}$

$= \{ab - ab^2 - a^2b + a^2b^2\}$

$= \{ab - ab - (ab)^2 + a^2b^2\}$ (using Lemma 3.2)

$= 0$

Hence proved.

Lemma 3.5

Let R be a weak commutative Boolean like near-ring. Let S be a commutative subset of Rwhich is multiplicatively closed. Define a relation N on $R \times S$ by $(r_1, s_1) \sim (r_2, s_2)$ iff there existsan element $s \in S$

Proof:

(i) Let $(r, s) \in R \times S$

Since $rs - rs = 0$

$\Rightarrow (rs - rs) = 0$ for all $t \in S$

Hence '∼' is reflexive.

(ii) Let$(r_1, s_1) \sim (r_2, s_2)$

Then there exists an element $s \in S$ such that

$$(r_1 s_2 - r_2 s_1) = 0$$

$$\Rightarrow (r_2 s_1 - r_1 s_2) = 0$$

such that $s(r_1 s_2 - r_2 s_1) = 0$. Then N is an equivalence relation.

Hence '∼' is symmetric.

(iii) Let $(r_1, s_1) \sim (r_2, s_2)$ and $(r_2, s_2) \sim (r_3, s_3)$

Then there exists $p, q \in S$ such that

$(r_1 s_2 - r_2 s_1) = 0$ and $(r_2 s_3 - r_3 s_2) = 0$

$\Rightarrow p s_3 (r_1 s_2 - r_2 s_1) = 0 = q s_1 (r_2 s_3 - r_3 s_2)$

$\Rightarrow (r_1 s_2 - r_2 s_1)_3 = 0 = (r_2 s_3 - r_3 s_2) s_1$ (R- weak commutative)

$\Rightarrow (r_1 s_2 - r_2 s_1)_3 = 0 = pq (r_2 s_3 - r_3 s_2) s_1$

$\Rightarrow (r_1 s_2 s_3 - r_2 s_1 s_3) = 0 = p (r_2 s_3 s_1 - r_3 s_2 s_1)$

$\Rightarrow pq\ (r_1 s_2 s_3 - r_2 s_1 s_3 + r_2 s_3 s_1 - r_3 s_2 s_1) = 0$

$\Rightarrow (r_1 s_3 s_2 - r_2 s_1 s_3 + r_2 s_1 s_3 - r_3 s_1 s_2) = 0$ (S is commutative)

$\Rightarrow (r_1 s_3 s_2 - r_3 s_1 s_2) = 0$

$\Rightarrow (r_1 s_3 - r_3 s_1)_2 = 0$

$\Rightarrow pq\ s_2 (r_1 s_3 - r_3 s_1) = 0$ (R is weak commutative)

$\Rightarrow (r_1 s_3 - r_3 s_1) = 0$ where $r = pq\ s_2 \in S$

$\Rightarrow (r_1, s_1) \sim (r_3, s_3)$

Hence '$\sim$' is transitive.

Hence the lemma.

Theorem 3.6

Let R be a weak commutative Boolean like near -ring. Let S be a commutative subset of R which is also multiplicatively closed. Define binary operation '$+$' and on $s^{-1}R$ as follows. $\frac{r_1}{s_1} + \frac{r_2}{s_2} = \frac{r_1 s_2 + r_2 s_1}{s_1 s_2}$ and $\frac{r_1}{s_1} . \frac{r_2}{s_2} = \frac{r_1 r_2}{s_1 s_2}$. Then $s^{-1} R$ is a commutative Boolean like near- ring with identity.

Proof:

Let us first prove that '$+$' and '$\cdot$' are well-defined.

Let $\frac{r_1}{s_1} = \frac{r_1^F}{s_1^F}$ and $\frac{r_2}{s_2} = \frac{r_2^F}{s_2^F}$, then there exists $t_1, t_2 \in S$ such that

$(r_1 s'_1 - r_1 s'_1) = 0$ ....(1)

and $t(r_2 s'_2 - r_2 s'_2) = 0$ ....(2)

Now, $t_1 t_2 [\ (r_1 s_2 + r_2 s_1) s'_1 s'_2 - (r'_1 s'_2 + r'_1 s'_2) s_1 s_2]\ )$

$= t_1 t_2 [\ r_1 s_2 s'_1 s'_2 + r_2 s_1\ s'_1 s'_2 - r'_1 s'_2 s_1 s_2 - r'_2 s'_1 s_1 s_2]$

$= t_1 t_2 [\ r_1 s'_1 s_2 s'_2 - r'_1 s_1 s_2 s'_2 + r_2 s'_2 s_1 s'_1 - r'_2\ s_2 s_1 s'_1]$ (S is commutative subset)

$= t_1t_2[(r_1s_1' - r_1's_1)s_2s_2' + (r_2s_2' - r_2's_2)s_1s_1']$

$= t_1t_2(r_1s_1' - r_1's_1)s_2s_2' + t_1t_2(r_2s_2' - r_2's_2)s_1s_1'$

$= t_1(r_1s_1' - r_1's_1)t_2s_2s_2' + t_2(r_2s_2' - r_2's_2)t_1s_1s_1'$ (R is weak commutative)

$= 0.t_2s_2s_2' + 0.s_1s_1't_1$

$= 0$

Hence $\frac{r_1s_2+r_2s_1}{s_1s_2} = \frac{r_1's_2'+r_2's_1'}{s_1s_2'}$

(i.e.) $\frac{r_1}{s_1} + \frac{r_2}{s_2} = \frac{r_1'}{s_1'} + \frac{r_2'}{s_2'}$

Hence ' + ' is well-defined.

From (2) We get,

$t_1t_2(r_1s_1' - r_1's_1)r_2s_2' = 0$

$t_1t_2(r_1s_1'r_2 - r_1's_1r_2)s_2' = 0$

$t_1t_2(r_1s_1'r_2s_2' - r_1's_1r_2s_2') = 0$

$t_1t_2(r_1r_2s_1's_2' - r_1'r_2s_1s_2') = 0$ (S is commutative subset)

$t_1t_2r_1r_2s_1's_2' - t_1t_2r_1'r_2s_1s_2' = 0$ ………….(3)

From (2) we get

$t_1t_2(r_2s_2' - r_2's_2)r_1's_1 = 0$

$t_1t_2r_1's_1(r_2s_2' - r_2's_2) = 0$ (R is weak commutative)

$t_1t_2(r_1's_1r_2s_2' - r_1's_1r_2's_2) = 0$

$t_1t_2(r_1's_1s_2'r_2 - r_1's_1s_2r_2') = 0$ (S is commutative)

$t_1t_2(r_1'r_2s_1s_2' - r_1'r_2's_1s_2) = 0$

$t_1t_2r_1'r_2s_1s_2' - t_1t_2r_1'r_2's_1s_2 = 0$ ……….(4)

(3)+(4) gives

$t_1t_2r_1r_2s_1's_2' - t_1t_2r_1'r_2's_1s_2 = 0$

$t_1t_2(r_1r_2s_1's_2' - r_1'r_2's_1s_2) = 0$

This means $\frac{r_1r_2}{s_1s_2} = \frac{r_1'r_2'}{s_1's_2'}$

Hence '·' is well-defined.

We note that

$$\frac{r_1}{s_1} + \frac{r_2}{s_2} = \frac{r_1s-2+r_2s_1}{s_1s_2} = \frac{(r_1+r_2)s}{s^2} \; (\therefore s_1 = s_2)$$
$$= \frac{r_1+r_2}{s}$$

Claim1-($S^{-1}R$,+)is an abelian group

Let $\frac{r_1}{s_1}, \frac{r_2}{s_2}, \frac{r_3}{s_3} \in S^{-1}R$

Now,

$$\frac{r_1}{s_1} + \left(\frac{r_2}{s_2} + \frac{r_3}{s_3}\right) = \frac{r_1}{s_1} + \left(\frac{r_2 s_3 + r_3 s_2}{s_2 s_3}\right)$$

$$= \frac{r_1 s_2 s_3 + (r_2 s_3 + r_3 s_2) s_1}{s_1 s_2 s_3}$$

$$= \frac{r_1 s_2 s_3 + (r_2 s_3 + r_3 s_2) s_1}{s_1 s_2 s_3}$$

$$= \frac{r_1 s_2 s_3 + r_2 s_3 s_1 + r_3 s_2 s_1}{s_1 s_2 s_3}$$

Also, $\left(\frac{r_1}{s_1} + \frac{r_2}{s_2}\right) + \frac{r_3}{s_3} = \left(\frac{r_1 s_2 + r_2 s_1}{s_1 s_2}\right) + \frac{r_3}{s_3}$

$$= \frac{(r_1 s_2 + r_2 s_1) s_3 + r_3 s_1 s_2}{s_1 s_2 s_3}$$

$$= \frac{r_1 s_2 s_3 + r_2 s_3 s_1 + r_3 s_1 s_2}{s_1 s_2 s_3}$$

$$\frac{r_1}{s_1} + \left(\frac{r_2}{s_2} + \frac{r_3}{s_3}\right) = \left(\frac{r_1}{s_1} + \frac{r_2}{s_2}\right) + \frac{r_3}{s_3}$$

So'+' is associative.

For any $\frac{r}{s} \in s^{-1}R$, we have

$$\frac{r}{s} + \frac{0}{2} = \frac{r+0}{s} = \frac{r}{s}$$

Also, $\frac{0}{s} + \frac{r}{s} = \frac{0+s}{s} = \frac{r}{s}$

Hence $\frac{0}{s}$ is the additive identity of $\frac{r}{s} \in s^{-1}R, \forall r \in R$

Clearly '+' is commutative.

Thus $(R,+)$is an abelian group.

Claim-2'· 'is associative.

Now$\frac{r_1}{s_1} . \left(\frac{r_2}{s_2} . \frac{r_3}{s_3}\right) = \frac{r_1}{s_1} . \left(\frac{r_2 r_3}{s_2 s_3}\right) = \frac{r_1 (r_2 r_3)}{s_1 (s_2 s_3)} = \frac{(r_1 r_2) r_3}{(s_1 s_2) s_3}$ (R is weak commutative)

$$= \left(\frac{r_1}{s_1} . \frac{r_2}{s_2}\right) . \frac{r_3}{s_3}$$

So '.' is associative.

Claim-3'·'is right distributive with respect to+.

Let $\frac{r_1}{s_1},\frac{r_2}{s_2},\frac{r_3}{s_3}\in S^{-1}R$

Now$\left(\frac{r_1}{s_1}+\frac{r_2}{s_2}\right)\cdot\frac{r_3}{s_3}=\left(\frac{r_1s_2+r_2s_1}{s_1s_2}\right)\cdot\frac{r_3}{s_3}$

$$=\frac{r_1s_2r_3+r_2s_1r_3}{s_1s_2s_3}$$

$$=\frac{s_2r_1r_3+s_1r_2r_3}{s_1s_2s_3}\text{(S is commutative sub set)}$$

$$=\frac{s_2r_1r_3}{s_1s_2s_3}+\frac{s_1r_2r_3}{s_1s_2s_3}$$

$$=\frac{r_1r_3}{s_1s_3}+\frac{r_2r_3}{s_2s_3}$$

$$=\frac{r_1}{s_1}.\frac{r_3}{s_3}+\frac{r_2}{s_2}.\frac{r_3}{s_3}$$

Hence right- distributive law is proved.

Claim- 4$S^{-1}R$ is a Boolean like near-ring.

It is already proved in claim-1that $2\left(\frac{r}{s}\right)=0$ for all $\frac{r}{s}\in S^{-1}R$

Let $a=\frac{r_1}{s_1}$and $b=\frac{r_2}{s_2}$be any two elements of $S^{-1}R$

Let $t\in S$ be any element

By lemma (3.4)$\Rightarrow(a-a^2)(b-b^2)t=0$

$$\Rightarrow\left(\frac{r_1}{s_1}-\frac{r_1^2}{s_1^2}\right)\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)t=0$$

$$\Rightarrow t\left(\frac{r_1}{s_1}-\frac{r_1^2}{s_1^2}\right)\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)=0\ \text{(S is commutative subset}$$

$$\Rightarrow t\left(\frac{r_1}{s_1}\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)-\frac{r_1^2}{s_1^2}\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)\right)=0$$

$$\Rightarrow t\frac{r_1}{s_1}\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)-t\frac{r_1^2}{s_1^2}\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)=0\ \text{(R is weak commutative)}$$

$$\Rightarrow t\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)\frac{r_1}{s_1}-t.\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)\frac{r_1^2}{s_1^2}=0$$

$$\Rightarrow t\left[\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)\frac{r_1}{s_1}-\left(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2}\right)\frac{r_1^2}{s_1^2}\right]=0$$

$$\Rightarrow t\left[\left(\frac{r_2s_2-r_2^2}{s_2^2}\right)\frac{r_1}{s_1}-\left(\frac{r_2s_2-r_2^2}{s_2^2}\right)\frac{r_1^2}{s_1^2}\right]=0$$

$$\Rightarrow t\left[\left(\frac{r_2s_2-r_2^2}{s_2^2}\right)\frac{r_1s_1}{s_1^2}-\left(\frac{r_2s_2-r_2^2}{s_2^2}\right)\frac{r_1^2}{s_1^2}\right]=0$$

$$\Rightarrow t\left[\left(\frac{r_2s_2r_1s_1-r_2^2r_1s_1}{s_2^2s_1^2}\right)-\left(\frac{r_2s_2r_1^2-r_2^2r_1^2}{s_2^2s_1^2}\right)\right]=0$$

$$\Rightarrow t\left[\left(\frac{r_2r_1s_2s_1-r_2^2r_1s_1}{s_2^2s_1^2}\right)-\left(\frac{s_2r_2r_1^2-r_2^2r_1^2}{s_2^2s_1^2}\right)\right]=0$$

$$\Rightarrow t\left[\frac{r_2r_1s_2s_1}{s_2^2s_1^2}-\frac{r_2^2r_1s_1}{s_2^2s_1^2}-\frac{s_2r_2r_1^2}{s_2^2s_1^2}+\frac{r_2^2r_1^2}{s_2^2s_1^2}\right]=0 \text{ (S is commutative subset)}$$

$$\Rightarrow t\left[\frac{r_2r_1}{s_2s_1}-\frac{r_2^2r_1}{s_2^2s_1}-\frac{r_2r_1^2}{s_2s_1^2}+\frac{r_2^2r_1^2}{s_2^2s_1^2}\right]=0$$

$\Rightarrow t(ba-b^2a-ba^2-b^2a^2)=0$

$\Rightarrow ba=b^2a-ba^2+b^2a^2$

$=b^2a-ba^2+(ba)^2$ (by lemma (3.1)

$\Rightarrow ba=ba(b+a-ba)$

Hence $S^{-1}R$is a Boolean like near-ring.

Claim-5 Multiplication in $S^{-1}R$ is commutative.

Let$\frac{r_1}{s_1},\frac{r_2}{s_2}$ be any two elements of $S^{-1}R$.

Then$\frac{r_1}{s_1}.\frac{r_2}{s_2}=\frac{r_1r_2}{s_1s_2}=\frac{r_1r_2s}{s_1s_2s}\forall s\in S$

$=\frac{sr_1r_2}{s_1s_2s}$ (S is commutative subset)

$=\frac{s(r_2r_1)}{s_1s_2s}$ (R is weak commutative)

$=\frac{(r_1r_2)s}{s_1s_2s}$ (S is commutative subset)

$=\frac{r_2}{s_2}.\frac{r_1}{s_1}$

Hence multiplication in $S^{-1}R$ is commutative.

Claim-6 Existence of multiplicative identity in $S^{-1}R$.

Let $\frac{r}{s} \in S^{-1}R$ be any element.

Then $\frac{r}{s}.\frac{s}{s} = \frac{rs}{ss} = \frac{r}{s}$

Then $\frac{s}{s}.\frac{r}{s} = \frac{sr}{ss} = \frac{r}{s}$

Hence $\frac{s}{s} \in S^{-1}R$ is the multiplicative identity of $S^{-1}R$

Thus $S^{-1}R$ is a commutative Boolean like near-ring with identity.

Theorem 3.7

$S^{-1}R$ is weak commutative near-ring

Proof:

Let $a=\frac{r_1}{s_1}, b = \frac{r_2}{s_2}, c = \frac{r_3}{s_3}$ be any three elements of $S^{-1}R$

Now $abc=\frac{r_1}{s_1}.\frac{r_2}{s_2}.\frac{r_3}{s_3} = \frac{r_1r_2r_3}{s_1s_2s_3}$

$= \frac{r_1r_3r_2}{s_1s_2s_3}$ (R is weak commutative)

$= \frac{r_1r_3r_2}{s_1s_3s_2}$(S is commutative)

$= \frac{r_1}{s_1}.\frac{r_2}{s_2}.\frac{r_3}{s_3}$

$=acb$

$\Rightarrow abc=acb \forall a,b,c \in S^{-1}R$

Hence $S^{-1}R$ is weak commutative near-ring.

Theorem 3.8

Let R be a weak commutative Boolean like near-ring. Let S be a commutative subset of R Which is multiplicatively closed. Let $0 \neq s \in S$.Define a map $f_s: R \to S^{-1}R \ as \ f_s(r) = \frac{rs}{s} \ \forall r \in R$.

Then f_s is near-ring monomorphism.

Proof:

Let $r_1,r_2 \in R$

Then $f_s(r_1 + r_2) = \frac{(r_1+r_2)s}{s}$

$= \frac{r_1s+r_2s}{s} = \frac{r_1s}{s} + \frac{r_2s}{s}$

$= f_s(r_1) + f_s(r_2)$

$f_s(r_1.r_2) = \frac{(r_1.r_2)s}{s}$

$= \frac{r_1 r_2 s^2}{s}$ (R is weak commutative)

$$= \frac{r_1 s^2 r_2}{s} = \frac{r_1 s s r_2}{s} = \frac{r_1 s (r_2 s)}{s}$$

$$= \frac{r_1 s}{s} \cdot \frac{r_2 s}{s}$$

$$= f_s(r_1) \cdot f_s(r_2)$$

Also, $f_s(r_1) = f_s(r_2)$

$$\Rightarrow \frac{r_1 s}{s} = \frac{r_2 s}{s}$$

$$\Rightarrow \frac{r_1 s}{s} - \frac{r_2 s}{s} = 0$$

$$\Rightarrow \frac{(r_1 - r_2)s}{s} = 0$$

$$\Rightarrow \frac{(r_1 - r_2)}{s} = 0$$

$$\Rightarrow \frac{r_1}{s} - \frac{r_2}{s} = 0$$

$$\Rightarrow \frac{r_1}{s} = \frac{r_2}{s}$$

Hence f_s is monomorphism.

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