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A NOTE ON P-IDEAL STRUCTURES IN TERNARY SEMIGROUPS

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ABSTRACT

In this paper we have defined left weakly p-prime, left weakly p-semiprime and left strongly p-prime ideals in ternary semigroup and proved that every left weakly p-semiprime ideal is left strongly p-prime ideal if it is strongly irreducible. It is proved that the concepts of left strongly p-prime ideal and left weakly p-prime ideals of a ternary semigroup are same. G-ternary set in a ternary semigroup is defined here and also it is shown that a G-ternary set in a ternary semigroup is left weakly p-prime ideal. Definitions of p-ideals and p-co-ideals are introduced in this paper with the property that intersection and product of any two p-co-ideals of a ternary semigroup is a p-co-ideal of a ternary semigroup. We also proved that every p-ideal is a p-co-ideal.

MATHEMATICAL SUBJECT CLASSIFICATION: 17A40, 20M17

KEYWORDS

Ternary semigroup, irreducible left ideals, strongly irreducible left ideals, prime ideals, semiprime ideals, co-ideals, left weakly semiprime ideals, left strongly prime ideals.

INTRODUCTION

Algebraic structure plays a prominent role in Mathematics with wide range of applications in Computer Science, Information Science, Engineering, Physics etc. In 1932, Lehmer introduced the concept of Ternary semigroup. He investigated certain ternary algebraic systems called triplexes which turn out to be commutative ternary semigroups. The concept of weakly prime and weakly semiprime ideals of ternary semigroups have been introduced and studied by Bindu.P et al. In this paper we have introduced and studied about structures of some types of p-ideals in Ternary semigroup.

MAIN RESULTS

P-PRIME IDEALS IN TERNARY SEMIGROUP

DEFINITION:2.1.1

Let T be a ternary semigroup and $S \subseteq T$. S is called *left(right, lateral) weakly p-prime ideal* if for any ideals A, B, C of T such that $ABC + P \subseteq S$ we have $A \subseteq S$ or $B \subseteq S$ or $C \subseteq S$.

DEFINITION:2.1.2

Let T be a ternary semigroup and $S \subseteq T$. S is called *left(right, lateral)strongly p-prime* if for any left ideals A, B, C of T such that $ABC \cap BCA \cap CAB + P \subseteq S$ we have $A \subseteq S$ or $B \subseteq S$ or $C \subseteq S$.

DEFINITION:2.1.3

Let Tbe a ternary semigroup and $S \subseteq T$. S is called *left(right, lateral) weakly p-semiprime*if for any left ideal A of T such that $\{0\} \neq A^3 + P \subseteq S$ we have $A \subseteq S$.

DEFINITION:2.1.4

(i) A ternary semigroup is called a *fully left weakly p-prime ternary semigroup* if all its ideals are *left weakly p-prime left ideals.*

(ii) A ternary semigroup is called a *fully left weakly p-semiprime ternary semigroup* if all its left ideals are *left weakly p-semiprime left ideals.*

THEOREM:2.1.5

Every strongly irreducible left weakly p-semiprime ideal of a ternary semigroup T is left strongly p-prime ideal.

Proof

First we take S to be left weakly p-semiprime ideal which is strongly irreducible. Let S be a left weakly p-semiprime ideal of a ternary semigroup T, then for any ideal V of T such that$\{0\} \neq V^3 + P \subseteq S$ we have $V \subseteq S$.Let A, B, C be any three left ideals of T such that $ABC \cap BCA \cap CAB + P \subseteq S$. $(A \cap B \cap C)^3 + P \subseteq ABC + P$, $(A \cap B \cap C)^3 + P \subseteq BCA + P$ and$(A \cap B \cap C)^3 + P \subseteq CAB + P$so $(A \cap B \cap C)^3 + P \subseteq ABC \cap BCA \cap CAB + P \subseteq S$. Since S is a left weakly p-semiprime ideal,$A \cap B \cap C + P \subseteq S$. As S is a strongly irreducible left ideal, either $A \subseteq S$ or $B \subseteq S$

or $C \subseteq S$. Thus S is a left strongly p-prime ideal. Hence proved.

THEOREM:2.1.6

The concepts of left strongly p-prime and left weakly p-prime ideals of T are the same.

Proof

Let S be a left weakly p-prime ideal of a ternary semigroup T. Then there exists any ideals ABC of T such that $ABC + P \subseteq S$ implies $A \subseteq S$or $B \subseteq S$ or $C \subseteq S$.Let A, B, C be any three ideals of T such that $ABC \cap BCA \cap CAB + P \subseteq S. ABC \cap BCA \cap CAB + P$ is a left ideal of T and $ABC \cap BCA \cap CAB + P \subseteq ABC \cap BCA \cap CAB + P$. By hypothesis, either $ABC + P \subseteq ABC \cap BCA \cap CAB + P$ or $BCA + P \subseteq ABC \cap BCA \cap CAB + P$ or $CAB + P \subseteq ABC \cap BCA \cap CAB + P$.So either $ABC + P \subseteq S$ or $BCA + P \subseteq S$ or $CAB + P \subseteq S$. Since S is left weakly p-prime ideal we have either $A \subseteq S$ or $B \subseteq S$ or $C \subseteq S$. That is S is left strongly p-prime ideal of T. Hence proved.

DEFINIITON:2.1.7

Let *T*be a ternary semigroup with *0*, then $a \in T$is called *left (lateral, right) zero element* if$0.0. a = 0. a. 0 = a. 0.0 = 0$.

DEFINITION:2.1.8

Let *T*be a ternary semigroup and $S \subseteq T. a \in T$ be a left (lateral, right) zero element. Then we define $(S, a) = \{x \in T \,/ xaa + p \in S\}$ as *G-ternary set* in T.

THEOREM:2.1.9

Let S be a left weakly p-prime ideal of a ternary semigroup T with zero element, then G-ternary set is a left weakly p-prime ideal of T for all $a \in T - S$.

Proof

Since S is a left weakly p-prime ideal of a ternary semigroup with $0 \in T$ and hence $0 \in (S, a)$. Therefore, $(S, a) \neq \varphi$.Let $x \in (S, a); b, c \in T$, we have, $(bcx)aa + p = b(cxa)a + p = bc(xaa) + p \in S$.In fact, $xaa + p \in S$ and S is an ideal of T. Thus, $bcx + p \in (S, a)$. Therefore, (S, a) is a left ideal of T.Let A, B, C be any three left ideals of T such that $ABP + P \subseteq (S, a)$ because Aaa, Baa, Caa are left ideals of T. This implies $Aaa \subseteq S$ or $Baa \subseteq S$ or $Caa \subseteq S$, which implies $A \subseteq (S, a)$ or $B \subseteq (S, a)$ or $C \subseteq (S, a)$.Therefore, (S, a) is a left weakly p-prime ideal of T.Hence proved.

P-LEFT (RIGHT, LATERAL) IDEALS.

DEFINITION:2.2.1

A non-empty subset S of a ternary semigroup T is said to be *p-left (right, lateral) ternaryideal*or *p-left (right, lateral) ideal* of T if $b, c \in T, a \in S$ implies $bca + p \in S$ $(abc + p \in S \,, bca + p \in S)$.

If a non-empty subset S of a ternary semigroup T is *p-left ideal*, *p-right ideal* and *p-lateral ideal*, we say it *p-ideal*of T.

THEOREM:2.2.2

The non-empty intersection of any two p-left ideals (right, lateral) of a ternary semigroup T is a p-left(right, lateral) ideal of T.

Proof

Let A, B be any two p-left ideals of T. Let $a \in A \cap B$ and $b, c \in T$, then $a \in A \cap B \Rightarrow a \in A$ and $a \in B$. $a \in A, b, c \in T, a$ is a p-left ideal of T which implies $bca + p \in A$. $a \in B, b, c \in T, b$ is a p-left ideal of T which implies $bca + p \in B$. $bca + p \in A, bca + p \in B$ which implies $bca + p \in A \cap B$. Therefore, $A \cap B$ is a p-left ideal of T. Hence proved.

COROLLARY:2.2.3

Finite intersection of any family of p-left (right, lateral) ideals of a ternary semigroup T is a p-left (right, lateral) ideal of T.

Proof

Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a family of p-left ideals of T and left $A = \bigcap_{\alpha\in\Delta} A_\alpha$.Let $a \in A, b, c \in T$. Now $a \in A, a \in \bigcap_{\alpha\in\Delta} A_\alpha \Rightarrow a \in A_\alpha$for each $\alpha \in \Delta$. $a \in A_\alpha, b, c \in T$. A_α is a p-left ideal of T implies $bca + p \in A_\alpha$ for all $\alpha \in \Delta$ implies $bca + p \in \bigcap_{\alpha\in\Delta} A_\alpha$implies $bca + p \in A$.Therefore, A is a p-left ideal of T. Hence proved.

THEOREM:2.2.4

The union of any two p-left (right,lateral) ideals of a ternary semigroup T is a p-left(right, lateral) ideal of T.

Proof

Let A_1, A_2 be any two p-left ideals of a ternary semigroup T. Let $A = A_1 \cup A_2$, clearly A is a non-empty subset of T. Let $a \in A, b, c \in T$. Now, $a \in A \Rightarrow a \in A_1 \cup A_2 \Rightarrow a \in A_1$ or $a \in A_2$.Suppose, $a \in A_1$, so $a \in A_1; b, c \in T. A_1$ is a p-left ideal of T. This implies $bca + p \in A_1 \subseteq A_1 \cup A_2 = A$. This implies, $bca + p \in A$.Suppose $a \in A_2$, so $a \in A_2; b, c \in T. A_2$ is a p-left ideal of T.This implies $bca + p \in A_2 \subseteq A_1 \cup A_2 = A$.This implies, $bca + p \in A$.Therefore, $a \in A, b, c \in T \Rightarrow bca + p \in A$ and hence A is a p-left ideal of T.

COROLLARY:2.2.5

Finite union of any family of p-left (right, lateral) ideals of a ternary semigroup T is a p-left ideal of T.

Proof

Let $\{A_\alpha\}_{\alpha\in\Delta}$be a family of p-left ideals of a ternary semigroup T. Let $A = \bigcup_{\alpha\in\Delta} A_\alpha$. Clearly, A is a non-empty subsets of T. Let $a \in A, b, c \in T, a \in A \Rightarrow a \in \bigcup_{\alpha\in\Delta} A_\alpha \Rightarrow$ $a \in A_\alpha$for some $\alpha \in \Delta$. $a \in A_\alpha, b, c \in T, A_\alpha$

is a p-left ideal of T, this implies, $bca + p \in A_\alpha \subseteq \bigcup_{\alpha \in \Delta} A_\alpha = A \Rightarrow bca + p \in A$.
Therefore A is a p-left ideal of T.

We have proved the Theorem 2.2.2, 2.2.4 and Corollary 2.2.3 and 2.2.5 for p-left ideals, in similar manner we can prove these results for p-right and p-lateral ideals of a ternary semigroup T.

P-CO IDEALS

DEFINITION:2.3.1

A non-empty subset S of a ternary semigroup T is called a*p-co ideal* if

(i) $a, b, c \in S$ implies $abc + p \in S$.

(ii) $a \in S, t \in T$ implies $at + p \in S$.

THEOREM:2.3.2

Let S, J be two p-co ideals of a ternary semigroup T, then

(i) $S \cap J$ is a p-co ideal of T

(ii) SJ is a p-co ideal of T

Proof

(i) Let $a, b, c \in S \cap J$, then $abc + p \in S$, since S, J is a p-co ideal and so $abc + p \in J$. Hence $abc + p \in S \cap J$. Let $a \in S \cap J$ and $r \in T$, then $at + p \in S$ and $at + p \in J$, since S, J is a p-co ideal of T. Thus $at + p \in S \cap J$. Hence $S \cap J$ is a p-co ideal of T.

(ii) Let $x = ab, y = a'b', z = a^*b^* \in SJ$, then $xyz + p = (ab)(a'b')(a * b *) + p = [(aa')(ab')(ba')(bb')][(a^*b^*)] + p = [(aa^*)(ab^*)(a'a^*)(a'b^*)][(aa^*)(ab^*)(b'a^*)$

$(b'b^*)][(ba^*)(bb^*)(a'a^*)(a'^{b^*})]$

$[(ba^*)(bb^*)(b'a^*)(b'^{b^*}) + p$

$= (aa^*)^2(ab^*)^2a'(a^*b^*)b'(a^*b^*)b(a^*b^*)a'(a^*b^*)b(a^*b^*)$

$+ p = (aa^*)^2(ab^*)^2a'b'ba'bb'(a^*b^*) + p$

$= (aa^*)^2(ab^*)^2a'^2b'^2b^2(a^*b^*) + p$

Since $(a^*b^*) \in SJ$ we have $xyz + p \in SJ$. Let $r \in T$, then $xr = (ab)r + p \in S$, since S is a p-co ideal of T. Hence SJ is a p-co ideal of T.

THEOREM:2.3.3

Every p-ideal is a p-co ideal.

Proof

Let S be a p-ideal of a ternary semigroup T and $a, b, c \in S$, then $abc + p \in S$. Let $a \in S$ and $t \in T$ then $at + p \in S$, since S ia a p-ideal of T. Thus S is a p-co ideal of T. Hence proved.

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