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A COMPARATIVE STUDY TO SOLVE DECISION MAKING FUZZY LINEAR PROGRAMMING PROBLEM WITH PENTAGONAL FUZZY NUMBERS BY RANKING TECHNIQUES

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ABSTRACT

In this paper, with the help of Pentagonal Fuzzy Numbers, a decision-making Fuzzy Linear Programming Problem (FLPP) is solved. In order to exhibit the Fuzzy Linear Programming Problem, a comparative study of two ranking techniques such as Subinterval Addition Method and Subinterval Average Method is analysed.

Keywords: Defuzzification, Pentagonal Fuzzy Numbers, Ranking Techniques, Fuzzy Linear Programming Problem

INTRODUCTION

Fuzzy sets are sets in mathematics that contain items with varying degrees of membership. Zadeh (1965) developed fuzzy sets as an extension of classical sets to describe non-statistical uncertainties. The use of fuzzy sets is common where there is uncertain or incomplete information. Numerous new mathematical theories addressing uncertainty and imprecision have been developed due to the introduction of fuzzy sets. One such thing is the fuzzy numbers. A fuzzy number is a generalisation of real number that does not refer to a single value but rather to a set of possible values. Linear programming is a special case of mathematical programming. It is a method to achieve the best outcome like profit of a sale. It is a technique for the optimization of a linear objective function, subject to linear equality and inequality constraints. It is widely utilised in business, industry, and other fields in addition to mathematics. The concept of decision making in a fuzzy environment was introduced by Bellman and Zadeh (1970). Tanaka and Asai. (1984) was the first to propose the concept of fuzzy linear programming. Zimmerman (1978) developed a solution to the fuzzy linear programming problem in the fuzzy environment. Fuzzy linear programming problem refers to a linear programming problem where all of the parameters are represented by fuzzy numbers. Fuzzification in linear programming problems comes under three categories: fuzzy linear programming problems with fuzzy constraints, fuzzy linear programming problems with fuzzy objective functions, and fuzzy linear programming problems with fuzzy objective functions and fuzzy constraints Verdegay (1984). Stephen Dinagar and Kamalanathan (2017) solved the fuzzy linear programming problem with fuzzy objective function and fuzzy constraints. Sundaresan *et al.* (2016) handled linear programming in resource management techniques.

Based on the ordering of the fuzzy numbers, it is easy to determine if a fuzzy number is greater or smaller. The ranking function, which converts each fuzzy integer into a real line, was used to do this. Lexicographic screening approach for ranking was developed by Wang *et al.* (2005). Area between centroid and its original point approach was developed by Wang and Lee (2008). Area technique, the improved PILOT ranking procedure and the SD of the PILOT procedure were all imported by Stephen Dinagar and Kamalanathan (2015). Subinterval Addition Method and Subinterval Average Method was proposed by Stephen Dinagar and Kamalanathan (2017) to defuzzify any fuzzy number and to solve the fuzzy linear programming problem. In this paper, with the help of pentagonal fuzzy numbers, we solve decision-making Fuzzy Linear Programming Problem (FLPP). In order to exhibit the fuzzy linear programming problem, we give a comparative study of two ranking techniques such as Subinterval Addition Method and Subinterval Average Method.

RANKING TECHNIQUES AND METHOD OF SOLVING FLPP

Stephen Dinagar and Kamalanathan (2017) have provided the procedure of the two types of ranking techniques, Subinterval Addition Method and Subinterval Average Method and also provided the algorithm for the two methods.

***Definition* 2.1:** [5] In a nonempty set X, a fuzzy set A is characterized by its membership function $\mu_A(x)$ whose values lies in $[0,1]$ for all x in X.

***Definition* 2.2 :** [5] A fuzzy number A is a fuzzy set of the real line with a normal, convex, and continuous membership function of bounded support. We denote the family of fuzzy numbers by F.

***Definition* 2.3:** [5] Pentagonal fuzzy number $A = (a_1, a_2, a_3, a_4, a_5)$ and its membership function is

$$\mu_A(x) = \begin{cases} 0 & , x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & , a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & , a_2 \le x \le a_3 \\ 1-\frac{1}{2}\left(\frac{x-a_3}{a_4-a_3}\right) & , a_3 \le x \le a_4 \\ \frac{1}{2}\left(\frac{a_5-x}{a_5-a_4}\right) & , a_4 \le x \le a_5 \\ 0 & , x > a_5 \end{cases}$$

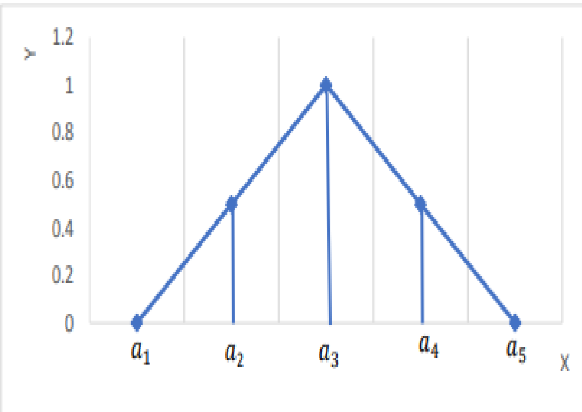

Figure 1: Pentagonal Fuzzy Number

Ranking Techniques 2.4: [5] A fuzzy number $(a_1, a_2, a_3, \ldots, a_n)$ defines on x-axis, each a_i is a real point, where the range of *i* varies from 1 to *n*.

The entire range is shuffled into intervals in the form (a_j, a_k) where $1 \le j \le k \le n$.

SUBINTERVAL ADDITION METHOD

Consider all the sums $a_j + a_k$ where $1 \le j \le k \le n$. The average of all this kind of possible additions defines the ranking function. When i takes the value n, ranking function $\Re(A^n) = \frac{2}{n}\sum_{i=1}^{n} a_i$

SUBINTERVAL AVERAGE METHOD

Find the average $(a_j + a_k)/2$ of each sub interval where $1 \le j \le k \le n$. The average of average of all this kind of intervals is our ranking function.

Therefore, $\Re(A^n) = \frac{1}{n}\sum_{i=1}^{n} a_i$

Method of Solving Fuzzy Linear Programming Problems 2.5: [5]

FLPP is specified as Maximize (or Minimize) $\tilde{Z} = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \cdots + \tilde{c}_n x_n$

Subject to $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i$, $i = 1,2,3,\ldots,m_0$, $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \ge \tilde{b}_i$, $i = m_0 + 1, \ldots, m$ and $x_j \ge 0\ \forall\ j = 1,2,3,\ldots,n$ where $x_j \in R$ and $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_j \in F(R)$ are fuzzy numbers A^i (in algorithm it is mentioned as A^t) where $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$

For example, in case of Triangular Fuzzy Number, we have

Ranking Techniques	**Intervals**	**Average of Upper and Lower Limits**	**Average of column (iii)**	**Ranking $\mathfrak{R}(A^3)$**	**In General, $\mathfrak{R}(A^n)$**
Subinterval Addition method	(a_1,a_2), (a_2,a_3), (a_1,a_3), (a_1,a_1), (a_2,a_2), (a_3,a_3)	(a_1+a_2), (a_2+a_3), (a_1+a_3), (a_1+a_1), (a_2+a_2), (a_3+a_3)	$\{[(a_1+a_2)]+[(a_2+a_3)]+[(a_1+a_3)]+[(a_1+a_1)]+[(a_2+a_2)]+[(a_3+a_3)]\}\div 6$	$\mathfrak{R}(A^3)=\frac{4(a_1+a_2+a_3)}{6}$	$\mathfrak{R}(A^n)=\frac{2}{n}\sum_{i=1}^{n}a_i$
Subinterval Average Method	(a_1,a_2), (a_2,a_3), (a_1,a_3), (a_1,a_1), (a_2,a_2), (a_3,a_3)	$[\frac{(a_1+a_2)}{2}]$, $[\frac{(a_2+a_3)}{2}]$, $[\frac{(a_1+a_3)}{2}]$, $[\frac{(a_1+a_1)}{2}]$, $[\frac{(a_2+a_2)}{2}]$, $[\frac{(a_3+a_3)}{2}]$	$\{\left[\frac{(a_1+a_2)}{2}\right]+\left[\frac{(a_2+a_3)}{2}\right]+\left[\frac{(a_1+a_3)}{2}\right]+\left[\frac{(a_1+a_1)}{2}\right]+\left[\frac{(a_2+a_2)}{2}\right]+\left[\frac{(a_3+a_3)}{2}\right]\}\div 6$	$\mathfrak{R}(A^3)=\frac{4(a_1+a_2+a_3)}{12}$	$\mathfrak{R}(A^n)=\frac{1}{n}\sum_{i=1}^{n}a_i$

ALGORITHM

Step 1: FLPP is Maximize (or Minimize) $\tilde{Z}=\sum_{j=1}^{n}\tilde{c}_j{}^{A^t}x_j$ subject to $\sum_{j=1}^{n}\tilde{a}_{ij}{}^{A^t}x_j\le\tilde{b}_i{}^{A^t}$, $i=1,2,3,\dots,m_0$, $\sum_{j=1}^{n}\tilde{a}_{ij}{}^{A^t}x_j\ge\tilde{b}_i{}^{A^t}$, $i=m_0+1,\dots,m$ and $x_j\ge 0\ for\ all\ j=1\ to\ n$ $and\ x_j\in R$, $\tilde{a}_{ij}{}^{A^t},\tilde{b}_i{}^{A^t},\tilde{c}_j{}^{A^t}\in F(R)$, $i=1\ to\ m$ and $j=1\ to\ n$

When t=3, $\tilde{a}_{ij}{}^{A^t},\tilde{b}_i{}^{A^t},\tilde{c}_j{}^{A^t}$ are triangular fuzzy numbers.

When t=4, $\tilde{a}_{ij}{}^{A^t},\tilde{b}_i{}^{A^t},\tilde{c}_j{}^{A^t}$ are trapezoidal fuzzy numbers.

When t=5, $\tilde{a}_{ij}{}^{A^t},\tilde{b}_i{}^{A^t},\tilde{c}_j{}^{A^t}$ are pentagonal fuzzy numbers and so on.

Step 2: By applying the ranking function form the relation I and II to the fuzzy numbers, $\tilde{a}_{ij}{}^{A^t},\tilde{b}_i{}^{A^t},\tilde{c}_j{}^{A^t}$, they can be defuzzified.

Step 3: Using the ranking function, the FLPP becomes a crisp valued LPP of the form:

Maximize (or Minimize) $\tilde{Z}=\sum_{j=1}^{n}R(\tilde{c}_j{}^{A^t})\,x_j$, subject to $\sum_{j=1}^{n}R\left(\tilde{a}_{ij}{}^{A^t}\right)x_j\ \ge,\le,=R(\tilde{b}_i{}^{A^t})$ and $x_j\ge 0\ for\ all\ j=1\ to\ n$ where $R(\tilde{a}_{ij}{}^{A^t}),R\left(\tilde{b}_i{}^{A^t}\right),R\left(\tilde{c}_j{}^{A^t}\right)\in R$, $i=1\ to\ m$ and $j=1\ to\ n$

Step 4: When the above equation is solved using the simplex method, we obtain the optimal solution.

SOLVING FLPP BY RANKING TECHNIQUES OF PENTOGONAL FUZZY NUMBERS – A COMPARATIVE STUDY

Here, we solve the above FLPP with objective function and constraints in pentagonal fuzzy numbers using simplex method. First, we defuzzify it and then simplex algorithm is applied to solve the FLPP.

A girl is running a YouTube channel where she uploads two types of videos, traditional drawing video and digital drawing video. For digital drawing, it will take (7, 10, 11, 22, 25) minutes to complete the drawing process and will take (0.4, 1.6, 2.1, 3.5, 4.9) minutes to complete the editing process. For traditional drawing, it will take (4, 9, 10, 12, 15) minutes to complete the drawing process and will take (1, 3.1, 5.6, 7.3, 8) minutes to complete the editing process. Totally, in a day, she spends (60, 100, 170, 200, 220) minutes for the drawing process and (22, 48, 50, 75, 80) minutes for the editing process. To be eligible for monetization in YouTube, she needs to increase the watch time (hours) of her channel. In a day, by uploading a single digital drawing video, she gains about (1, 2.2, 2.7, 4.1, 5) watch time every hour and by uploading a single traditional drawing video, she gains about (1.1, 3.6, 4, 5, 6.3) watch time every hour. How many of each video should she upload every day to increase the watch time of her channel, thus increasing her chance of monetization?

Solution: Let x_1 be the number of digital drawing videos and x_2 be the number of traditional drawing videos to be uploaded per day to increase the watch time.

Then, the mathematical form of the given problem will be

Maximize $Z = \tilde{c}_1 x_1 + \tilde{c}_2 x_2$ subject to $\tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 \leq \tilde{b}_1, \tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 \leq \tilde{b}_2$ and $x_1, x_2 \geq 0$ where $\tilde{a}_{11} = (7,10,11,22,25)$, $\tilde{a}_{12} = (4,9,10,12,15)$, $\tilde{a}_{21} = (0.4,1.6,2.1,3.5,4.9)$, $\tilde{a}_{22} = (1,3.1,5.6,7.3,8)$, $\tilde{b}_1 = (60,100,170,200,220)$, $\tilde{b}_2 = (22,48,50,75,80)$, $\tilde{c}_1 = (1,2.2,2.7,4.1,5)$, $\tilde{c}_2 = (1.1,3.6,4,5,6.3)$

SUBINTERVAL ADDITION METHOD

First let us defuzzify the numbers using the relation $\boldsymbol{\Re(A^n) = \frac{2}{n}\sum_{i=1}^{n} a_i}$

The given FLPP becomes Maximize $Z = 6x_1 + 8x_2$ subject to $30x_1 + 20x_2 \leq 300$, $5x_1 + 10x_2 \leq 110$ and $x_1, x_2 \geq 0$

Solving using simplex method, we get Max Z=96, $x_1 = 4, x_2 = 9$

Therefore, by uploading 4 digital drawing videos and 9 traditional drawing videos per day, she gains about 96 watch time per hour to increase her chance of monetization.

SUBINTERVAL AVERAGE METHO

First let us defuzzify the numbers using the relation $\mathfrak{R}(A^n) = \frac{1}{n}\sum_{i=1}^{n} a_i$

The given FLPP becomes Maximize $Z = 3x_1 + 4x_2$ subject to $15x_1 + 10x_2 \leq 150$, $\frac{5}{2}x_1 + 2x_2 \leq 55$ and $x_1, x_2 \geq 0$

Solving using simplex method, we get Max Z=48, $x_1 = 4, x_2 = 9$

Therefore, by uploading 4 digital drawing videos and 9 traditional drawing videos per day, she gains about 48 watch time per hour to increase her chance of monetization.

COMPARATIVE STUDY

The solution of the LPP $x_1 = 4$ and $x_2 = 9$ does not change by applying the above two different ranking techniques. In Subinterval Addition Method the objective function value is 96. It is reduced to 48 in Subinterval Average Method. This happens because the defuzzified value that we got by using the Subinterval Addition Method is twice the value that we got by using Subinterval Average Method.

CONCLUSION

An application of decision making FLPP with pentagonal fuzzy numbers was solved. A comparative study has been made by considering an example by applying the proposed two techniques. Thus, the Subinterval Addition Method is considered as the best method because the resulting objective function value is closure to corresponding value of early crisp problem.

REFERENCES

1. Bellman R and Zadeh LA (1970). Decision making in a fuzzy environment, *Management Science* 17:141-164.
2. Stephen Dinagar D and Kamalanathan S (2015). A Note on Maximize Fuzzy Net Present Value with New Ranking, *Intern. J. Fuzzy Mathematical Archive*.7(1):63-74.
3. Stephen Dinagar D and Kamalanathan S (2015). A Method for Ranking of Fuzzy Numbers Using New Area Method, *Intern. J. Fuzzy Mathematical Archive* 9(1):61-71.
4. Stephen Dinagar D, Kamalanathan S and Rameshan N (2016). A revised approach of PILOT ranking procedure of fuzzy numbers, *Global Journal of Pure and Applied Mathematics.* 12(2):309-313.
5. Stephen Dinagar D and Kamalanathan S (2017a). Solving Fuzzy Linear Programming Problem Using New Ranking Procedures of Fuzzy Numbers, *International Journal of Applications of Fuzzy Sets and Artificial Intelligence* 7 :281-292.

6. Stephen Dinagar D, Kamalanathan S and Rameshan N (2017b). Subinterval Average Method for Ranking of Linear Fuzzy Numbers, *International Journal of Pure and Applied Mathematics*,114(6):119-130.
7. Stephen Dinagar D, Kamalanathan S and Rameshan N (2017). Subinterval Addition Method for Ranking of Linear Fuzzy Numbers, *International Journal of Pure and Applied Mathematics*; 115(9):159-169.
8. Sundaresan V, Ganapathy Subramanian KS and Ganesan K (2016). Linear Programming in Resource Management Techniques, pp.1.59-1.89, A.R.S. Publications, Chennai, India.
9. Tanaka H and Asai K (1984). Fuzzy Linear Programming Problems with fuzzy numbers, *Fuzzy Sets and Systems;* 13:1-10.
10. Verdegay JL (1984). A dual approach to solve the fuzzy linear programming problem, *Fuzzy Sets and Systems*, 14:131-141.
11. Wang ML, Wang HF and Lung LC (2005). Ranking Fuzzy Number based on Lexicographic Screening Procedure, *International Journal of Information Technology and Decision Making*, 4: 663-678.
12. Wang YJ and Lee HS (2008). The Revised Method of Ranking Fuzzy Numbers with an Area between the Centroid and Original Points, Computers and Mathematics with Applications, 55(9): 2033-2042.
13. Zadeh LA (1965). Fuzzy Sets, *Information and Control*, 8:338-353.
14. Zimmerman HJ (1978). Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, 1:45-55.