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NORDHAUS-GADDUM TYPE RELATIONS ON CLOSED SUPPORT STRONG EFFICIENT DOMINATION NUMBER OF SOME STAR RELATED GRAPHS UNDER ADDITION AND MULTIPLICATION

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ABSTRACT

Let G = (V, E) be a simple graph with p vertices and q edges. Let S be a γ_{se}- set of G. Let v ϵ S.A closed support strong efficient domination number of v under addition is defined by $\sum_{u\epsilon N[v]} \deg u$ and it is denoted by supp $\gamma_{se}^{+}[v]$. A closed support strong efficient domination number of G under addition is defined by $\sum_{v\epsilon S}$ supp $\gamma_{se}^{+}[v]$ and it is denoted by supp $\gamma_{se}^{+}[G]$. A closed support strong efficient domination number of v under multiplication is defined by $\prod_{u\epsilon N[v]} \deg u$ and it is denoted by supp $\gamma_{se}^{\times}[v]$. A closed support strong efficient domination number of G under multiplication is defined by $\prod_{v\epsilon S}$ supp $\gamma_{se}^{\times}[v]$ and it is denoted by supp $\gamma_{se}^{\times}[G]$. In this paper, Nordhaus-Gaddum type relations on closed support strong efficient domination number of some star related graphs under addition and multiplication is studied.

Key words: Closed support of a graph under addition, Closed support of a graph under multiplication, Nordhaus-Gaddum type relations, Strong efficient dominating sets, Strong efficient domination number.

INTRODUCTION

In this paper, only finite, undirected and simple graphs are considered. Let G = (V, E) be a graph with p vertices and q edges. The degree of any vertex u in G is the number of edges incident with u and is denoted by deg u. A vertex of degree 0 in G is called an isolated vertex. The complement $\bar{G}$ of a graph *G* has V(G) as its vertex set and two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in *G*.

A subset S of V(G) of a graph G is called a dominating set of G if every vertex in V(G)\S is adjacent to a vertex in S (Hayness *et al.*,1998). The concept of strong (weak) efficient domination in graphs was introduced by Meena *et.al.*, (2013) and further studied by Murugan and Meena (2016) and Murugan (2019). Nordhaus – Gaddum type relations on strong efficient dominating sets are studied in (Murugan and Meena and Murugan, 2019). A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every point $v \in V(G)$, we have $|N_s[v] \cap S| = 1$ ($|N_W[v] \cap S| = 1$), where $N_s(v) = \{u \in V(G); uv \in E(G), \text{degu}\, u \geq \deg v\}$ and $N_s[v] = N_s(v) \cup \{v\}$($N_w(v) = \{u \in V(G); uv \in E(G), \deg u \leq \deg v\}$ and $N_w[v] = N_w(v) \cup \{v\}$). The minimum cardinality of a strong (weak) efficient dominating set is called strong (weak) efficient domination number and is denoted by $\gamma_{se}(G)(\gamma_{we}(G))$. A graph G is strong efficient if there exists a strong efficient dominating set of G.

Balamurugan *et.al* (2019) introduced the concept of closed support of a graph under addition and closed support of a graph under multiplication. Let G = (V, E) be a graph. A closed support of a point v under addition is defined by $\sum_{u\in N[v]} \text{degu}$ and it is denoted by $supp[v]$. A closed support of a graph, G under addition is defined by $\sum_{v\in V(G)} \text{supp}[v]$ and it is denoted by $supp[G]$. A closed support of a point, v under multiplication is defined by $\prod_{u\epsilon N[v]} \deg u$ and it is denoted by $\text{mult}[v]$. An open support of a graph, G under multiplication is defined by $\prod_{v\in V(G)} \text{mult}\,[v]$ and it is denoted by $\text{mult}[G]$.

Meena and Murugan (2022) introduced the concept of closed support strong efficient domination number of a graph under addition and multiplication. In this paper, Nordhaus- Gaddum type relations on closed support strong efficient domination number of some star related graphs under addition and multiplication is studied.

For all graph theoretic terminologies and notations, Harary (1969) is followed. The following definitions and results are necessary for the present study.

Definition1.1 [8]

Let G = (V, E) be a strong efficient graph. Let S be a γ_{se}- set of G. Let v ϵ S. A closed support strong efficient domination number of v under addition is defined by $\sum_{u\epsilon N[v]} \deg u$ and it is denoted by supp $\gamma_{se}^{+}[v]$.

Definition 1.2 [8]

Let G = (V, E) be a strong efficient graph. Let S be a γ_{se}- set of G. Let v ϵ S. A closed support strong efficient domination number of G under addition is defined by $\sum_{v\epsilon S}$ supp $\gamma_{se}^{+}[v]$ and it is denoted by supp$\gamma_{se}^{+}[G]$.

Definition1.3 [8]

Let = G (V, E) be a strong efficient graph. Let S be a γ_{se}- set of G. Let v ϵ S. A closed support strong efficient domination number of v under multiplication is defined by $\prod_{u\epsilon N[v]} \deg u$ and it is denoted by supp $\gamma_{se}^{\times}[v]$.

Definition 1.4 [8]

Let G =(V, E) be a strong efficient graph. Let S be a γ_{se}- set of G. Let v ϵ S. A closed support strong efficient domination number of G under multiplication is defined by $\prod_{v\epsilon S}$ supp $\gamma_{se}^{\times}[v]$ and it is denoted by supp$\gamma_{se}^{\times}[G]$.

Definition 1.5 [18]

The line graph L(G) of G is the graph whose vertex set is E(G) in which two vertices are adjacent if and only if they are adjacent in G.

Definition 1.6 [3]

The jump graph J(G) of G is the graph whose vertex set is E(G) in which two vertices are adjacent if and only if they are nonadjacent in G.

Definition 1.7 [13]

The paraline graph PL(G) is a line graph of the subdivision graph of G.

Definition 1.8

Let G be a simple graph. The semi-total point graph $T_2(G)$ is the graph whose vertex set is V(G) ∪ E(G), where two vertices are adjacent if and only if

(i) they are adjacent vertices of G or
(ii) one is a vertex of G and the other is an edge of G incident with it.

Definition 1.9

Let G be a simple graph. The semi-total line graph $T_1(G)$is the graph whose vertex set is V(G) ∪ E(G) where two vertices are adjacent if and only if

(i) they are adjacent edges of G or
(ii) one is a vertex of G and the other is an edge of G incident with it.

Definition 1.10

Let G be a simple graph. The total graph T(G) is the graph whose vertex set is V(G) ∪ E(G), where two vertices are adjacent if and only if

(i) they are adjacent vertices of G or

(ii) they are adjacent edges of G or

(iii) one is a vertex of G and the other is an edge of G incident with it.

Definition 1.11

Let G be a simple graph. The quasi-total graph P(G) is the graph whose vertex set is V(G) ∪ E(G), where two vertices are adjacent if and only if

(i) they are non adjacent vertices of G or

(ii) they are adjacent edges of G or

(iii) one is a vertex of G and the other is an edge of G incident with it.

Definition 1.12

Let G be a simple graph. The quasi-vertex total graph Q(G) is the graph whose vertex set is V(G) ∪ E(G), where two vertices are adjacent if and only if

(i) they are adjacent vertices of G or

(ii) they are non adjacent vertices of G or

(iii) they are adjacent edges of G or

(iv) one is a vertex of G and the other is an edge of G incident with it.

Definition 1.13 [4]

Bistar $D_{m,n}$ is the graph obtained from K_2 by joining m pendant edges to one end vertex of K_2 and n pendant edges to the other end of K_2. The edge K_2 is called the central edge of $D_{m,n}$ and the vertices of K_2 are called the central vertices of K_2

Definition 1.14 [16]

A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing all edges incident to v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.15[7]

For a graph G, the complementary prism, denoted by $G\bar{G}$, is formed from a copy of G and a copy of $\bar{G}$ by adding a perfect matching between corresponding vertices.

Previous results 1.16 [8,12]

a. Let G = $P_{3n,}\ n \in N$. Then

i. suppγ_{se}^{+}(G) = 4n – 2

ii. supp$\gamma_{se}^{\times}$(G) = 4^{n-1}

b. Let G = $P_{3n+1}, n \in N$. Then

i. supp γ_{se}^{+}(G) = 4n+ 1

ii. supp $\gamma_{se}^{\times}$(G) = 4^{n}

c. Let G = $K_n,\ n \in N$. Then

i. suppγ_{se}^{+}(G) = $(n-1)^2$

ii. supp$\gamma_{se}^{\times}$(G) = $(n-1)^{n-1}$

d. Let G = $K_{1,n}, n \in N$. Then

i. supp γ_{se}^{+}(G) = n

ii. supp $\gamma_{se}^{\times}$(G) =1

e. Let G = $D_{m,n}$, m, n ∈ N. Then

i. supp γ_{se}^{+}(G) = $m + (n+1)^2$, if m ≥ n

ii. supp$\gamma_{se}^{\times}$(G) = $(n+1)^{n+1}$, if m ≥ n

f. $D_{1,s[v]}$, s ≥ 1 is strong efficient.

g . $D_{1,s[u_1]}$, s ≥ 1 is strong efficient.

h. $D_{r,s[u,v]}$, r, s ≥ 1 is strong efficient.

Remark 1.17

Let G = (V, E) be a strong efficient graph. If v is an isolated point, then supp $\gamma_{se}^{+}[v]$ = supp $\gamma_{se}^{\times}[v]$ = 0 and also supp $\gamma_{se}^{\times}[G]$ = 0. If G = $\overline{K_n}$, then supp $\gamma_{se}^{+}[G]$ = supp $\gamma_{se}^{\times}[G]$ = 0.

MAIN RESULTS

Theorem 2.1: Let G = $K_{1,n}$, n ∈ N and G' = L($K_{1,n}$) = K_n.Then

i. supp $\gamma_{se}^{+}[G]$ + supp $\gamma_{se}^{+}[G']$ = 2n + $(n-1)^n$

ii. supp $\gamma_{se}^{\times}[G]$ + supp $\gamma_{se}^{\times}[G']$ = n + $(n-1)^n$

Proof: The theorem follows immediately from the previous results 1.16 (c) & 1.16 (d)

Theorem 2.2: Let G = $K_{1,n}$ n ∈ N and G' = J($K_{1,n}$) =$\overline{K}_n$.Then

i. supp$\gamma_{se}^{+}[G]$+ supp $\gamma_{se}^{+}[G']$=2n

ii. supp$\gamma_{se}^{\times}[G]$+ supp$\gamma_{se}^{\times}[G']$ = n

Proof: The theorem follows immediately from the previous result 1.16 (d) & Remark 1.17.

Theorem 2.3: Let G = $K_{1,n}$, n ∈ N and G' = PL($K_{1,n}$). Then

i. supp$\gamma_{se}^{+}[G]$+ supp $\gamma_{se}^{+}[G']$=2n (n+1)

ii. supp $\gamma_{se}^{\times}[G]$ + supp $\gamma_{se}^{\times}[G']$ = n + $n^{n+(n-1)}$

Proof: Let G = $K_{1,n}$, n∈N. Let V(G) = $\{v, v_1, v_2, \dots, v_n\}$ and E(G) = $\{e_1, e_2, \dots, e_n\}$ where $e_i = vv_i$; 1 ≤i≤ n Let V(S(G)) = $\{v, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$and E(G) =$\{e_1, e_2, \dots, e_{2n}\}$where $e_i = vu_i$,1 ≤i≤ n and $e_{n+i} = u_i v_i$, 1 ≤i≤n. Let G' = PL ($K_{1,n}$).V(G') = $\{e_1, e_2, \dots, e_{2n}\}$, deg e_i = n,1 ≤i≤ n and deg e_i = 1, n + 1 ≤i≤ 2n. Then S_i = $\{e_i, e_{n+k}$/ 1≤ k ≤ n, k ≠i , 1 ≤i≤ n} are distinct γ_{se}- sets of G' [13].

i. supp $\gamma_{se}^{+}[G]$ = 2n. Consider S_1 for G'(Proof is similar for other sets).

supp$\gamma_{se}^{+}[e_1]$=$\sum_{v\in N[e_1])}$ deg v = $\sum_{i=1}^{n}$ deg e_i + deg e_{n+1} = (n) n + 1.

For 2 ≤ k ≤ n, supp $\gamma_{se}^{+}[e_{n+k}]$ = deg e_k = n+1. Therefore supp $\gamma_{se}^{+}[G']$ = $\sum_{v\in S_1}$ supp $\gamma_{se}^{+}[v]$ = supp γ_{se}^{+} [e_1] + $\sum_{k=1}^{n}$ supp $\gamma_{se}^{+}[e_{n+k}]$ = (n)n + 1+(n – 1)(n +1)= $2n^2$

Hence suppγ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$ = 2n + $2n^2$ = 2n (n+1)

ii. supp $\gamma_{se}^{\times}$ [G] = n. Consider S_1 for G'(Proof is similar for other sets).

supp $\gamma_{se}^{\times}[e_1] = \prod_{v\in N[e_1]} \deg v = \prod_{i=1}^{n} \deg e_i \times \deg e_{n+1} = n^n$. For 2≤ k ≤ n, supp $\gamma_{se}^{\times}[e_{n+k}] = \deg e_k$ = n. Therefore supp $\gamma_{se}^{\times}[G'] = \prod_{v\in S_1}$ supp $\gamma_{se}^{\times}[v]$ = supp $\gamma_{se}^{\times}[e_1] \times \prod_{k=2}^{n}$ supp $\gamma_{se}^{\times}[e_{n+k}] = n^n \times n^{n-1}$. Hence supp $\gamma_{se}^{\times}$(G) + supp $\gamma_{se}^{\times}(G')$ = n + $n^n \times n^{n-1}$= n + $n^{n+(n-1)}$

Theorem 2.4: Let G = $K_{1,n}$, n ∈N and $G' = T_2[K_{1,n}]$. Then

i. suppγ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$ = 8n

ii. supp $\gamma_{se}^{\times}$ [G]+ supp $\gamma_{se}^{\times}[G']$ = n + $2n \times 4^n$.

Proof: Let G = $K_{1,n}$, n ∈ N. Let V(G) = $\{v, v_1, v_2, \dots, v_n\}$ and E(G) = $\{e_1, e_2, \dots e_n\}$ where $e_i = vv_i$. Let $G' = T_2(K_{1,n})$. Then V(G') = $\{v, v_1, v_2, \dots, v_{n,} e_1, e_2, \dots, e_n\}$, degv = 2n, deg e_i = deg v_i = 2, 1 ≤i≤ n.

i. suppγ_{se}^{+}[G] = 2n. When n =1, $G' = K_3$. supp$\gamma_{se}^{+}[G']$ = 6. Suppose n ≥ 1. {v} is the unique γ_{se}- set of G'. supp$\gamma_{se}^{+}[G']$ = supp [v] = $\sum_{u\in N[v]} \deg u$ = degv+$\sum_{i=1}^{n}$[deg e_i + deg v_i = 6n. Hence supp γ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$ = 8n.

ii. supp$\gamma_{se}^{\times}$[G] = n. When n =1, $G' = K_3$. supp$\gamma_{se}^{\times}[G']$ =8. Suppose n ≥ 1. {v} is the unique γ_{se}- set of G'[13]. supp$\gamma_{se}^{\times}[G']$ = supp $\gamma_{se}^{\times}$ [v] = $\prod_{u\in N[v]} \deg u$ =deg v × $\prod_{i=1}^{n}$[deg $e_i \times$ deg v_i] = $2n \times 4^n$. Hence supp $\gamma_{se}^{\times}$(G) + supp $\gamma_{se}^{\times}(G')$ = n + $2n \times 4^n$.

Theorem 2.5: Let G = $K_{1,n}$, n ∈N and $G' = T_1(K_{1,n})$. Then

i. supp γ_{se}^{+} [G]+ supp $\gamma_{se}^{+}[G']$ = $2n^2 +$ 5n – 1

ii. supp $\gamma_{se}^{\times}$ [G]+ supp $\gamma_{se}^{\times}[G']$ = n + n$(n+1)^{2n-1}$

Proof: Let G = $K_{1,n}$, n ∈ N. Let V(G) = $\{v, v_1, v_2, \dots, v_n\}$ and E(G) = $\{e_1, e_2, \dots, e_n\}$ where $e_i = vv_i$. Let $G' = T_1(K_{1,n})$. Then V(G') = $\{v, v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_n\}$,deg v = n, deg e_i = n + 1 and deg v_i = 1, 1 ≤i≤ n. Then S_i = { e_i, v_j/1 ≤ j ≤ n and j ≠i}, 1 ≤i≤ n areγ_{se}-sets of G'[13].

i. supp γ_{se}^{+} [G] = 2n. Consider S_1 for G' (Proof is similar for other sets). supp$\gamma_{se}^{+}[e_1]$ =$\sum_{v\in N[e_1]} \deg v = \sum_{i=1}^{n} \deg e_i$ + deg v + degv_1 = n(n + 1) + n + 1 = $(n+1)^2$. supp$\gamma_{se}^{+}[v_i]$ = degv_i + deg e_i = n + 2, 2≤i≤ n. Therefore supp$\gamma_{se}^{+}[G']$ =supp$\gamma_{se}^{+}[e_1]$ + $\sum_{i=2}^{n}$ supp $\gamma_{se}^{+}[v_i]$ = $(n+1)^2$ + (n – 1)(n + 2) = $2n^2$+ 3n – 1.

Hence suppγ_{se}^{+}[G]+supp $\gamma_{se}^{+}[G']$=$2n^2$+5n– 1

ii. supp$\gamma_{se}^{\times}$[G] = n. Consider S_1 for G' (Proof is similar for other sets). supp$\gamma_{se}^{\times}[e_1]$ = $\prod_{v\in N[e_1]} \deg v = \prod_{i=1}^{n} \deg e_i$ × deg v × deg v_1 = $(n+1)^n$ × n × 1 = n$(n+1)^n$.

supp$\gamma_{se}^{\times}[v_i]$ =degv_i × deg e_i = n + 1, 2≤i≤ n. Therefore supp $\gamma_{se}^{\times}[G']$ = supp $\gamma_{se}^{\times}[e_1]$ × $\prod_{i=2}^{n}$ supp $\gamma_{se}^{+}(v_i)$ = n $(n+1)^n$ × $(n+1)^{n-1}$= n$(n+1)^{2n-1}$

Hence supp $\gamma_{se}^{\times}$(G) + supp $\gamma_{se}^{\times}(G')$ = n + n$(n+1)^{2n-1}$

Theorem 2.6: Let G = $K_{1,n}$, n ∈N and G' = T($K_{1,n}$). Then

i. supp γ_{se}^{+}[G]+supp $\gamma_{se}^{+}[G']$ = n(n + 7).

i. supp $\gamma_{se}^{\times}$ [G]+ supp $\gamma_{se}^{\times}[G']$ = n + 2n$[2(n+1)]^n$.

Proof: Let G = $K_{1,n}$, n ∈ N. Let V(G) =$\{v, v_1, v_2, \dots, v_n\}$ and E(G) = $\{e_1, e_2, \dots, e_n\}$ where $e_i = vv_i$. Let G' = T($K_{1,n}$). Then V(G') = $\{v, v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_n\}$. deg v = 2n, deg e_i = n + 1, 1 ≤i≤ n and deg v_i = 2, 1 ≤i≤ n.

ii. suppγ_{se}^{+}[G] = 2n. When n =1, G'= K_3. supp $\gamma_{se}^{+}[G']$ = 6. Suppose n ≥ 1. S = {v} is the unique γ_{se} - set of G' [13]. Hence supp$\gamma_{se}^{+}[G']$ = supp $\gamma_{se}^{+}[v]$ = $\sum_{u\in N[v]}$ deg u = deg v + $\sum_{i=1}^{n}$ deg e_i + $\sum_{i=1}^{n}$ deg v_i =n(n + 5). Hence supp γ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$= n(n + 7).

iii. supp$\gamma_{se}^{\times}$[G] = n. When n =1, G'= K_3. supp $\gamma_{se}^{\times}[G']$= 8. Suppose n ≥ 1. S ={v} is the unique γ_{se} - set of G'. supp $\gamma_{se}^{\times}[G']$ =supp $\gamma_{se}^{\times}$ [v] = degv × $\prod_{u\in N[v]}$ deg u = degv× $\prod_{i=1}^{n}[degv_i \times \text{deg } e_i]$ = 2n$[2(n+1)]^n$. Hence supp $\gamma_{se}^{\times}$[G]+ supp $\gamma_{se}^{\times}[G']$ = n + 2n$[2(n+1)]^n$.

Remark 2.7: Let G = $K_{1,n}$, n = 1and G' = P($K_{1,n}$) = P_3 Then supp γ_{se}^{+} [G] + supp $\gamma_{se}^{+}[G']$ = 6 and supp[$\gamma_{se}^{\times}$(G] + supp $\gamma_{se}^{\times}[G']$ = 3.

Theorem 2.8: Let G = $K_{1,n}$, n ∈N and G' = Q($K_{1,n}$). Then

i. supp γ_{se}^{+}[G]+supp $\gamma_{se}^{+}[G']$ =n(2n + 6)

ii. supp $\gamma_{se}^{\times}$[G] + supp $\gamma_{se}^{\times}[G']$ = n + $2n(n+1)^{2n}$

Proof: Let G = $K_{1,n}$, n ∈ N. Let V(G) = $\{v, v_1, v_2, \dots, v_n\}$ and E(G) = $\{e_1, e_2, \dots, e_n\}$ where $e_i = vv_i$. Let G'denote the graph Q($K_{1,n}$). Then we have V(G') = $\{v, v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_n\}$ degv = 2n, dege_i = deg v_i = n + 1,1 ≤i≤ n.

i. supp γ_{se}^{+}(G) = 2n. When n =1, G'= K_3. supp $\gamma_{se}^{+}(G')$ = 6. Suppose n ≥ 1. {v} is the unique γ_{se} - set of G' [13]. Therefore supp $\gamma_{se}^{+}(G')$ = supp$\gamma_{se}^{+}(v)$ = $\sum_{u\in N(v)}$ deg u = degv+ $\sum_{i=1}^{n}$ deg e_i+ $\sum_{i=1}^{n}$ deg v_i = 2n+ n(n + 1) + n(n + 1) = $2(n^2+2n)$.

Hence supp γ_{se}^{+}(G) + supp $\gamma_{se}^{+}(G')$ = $2n^2$ + 6n = n(2n + 6).

ii. supp$\gamma_{se}^{\times}$(G) = n. When n =1, G'= K_3. supp$\gamma_{se}^{\times}(G')$ = 8. Suppose n ≥ 1. {v} is the unique γ_{se}- set of G'. supp $\gamma_{se}^{\times}(G')$ = supp $\gamma_{se}^{\times}$ (v) = $\prod_{u\in N(v)} \deg(u)$ = degv × $\prod_{i=1}^{n}[\deg v_i \times \deg e_i]$ = $2n(n+1)^n \times (n+1)^n$ =2n $(n+1)^{2n}$ .Hence supp $\gamma_{se}^{\times}$ (G) + supp$\gamma_{se}^{\times}(G')$ = n +$2n(n+1)^{2n}$

Remark 2.9: Let G = $K_{1,n}$, n = 1. G' = $G\bar{G}$ = $K_{1,n}\overline{K}_{1,n}$ = P_4. Then supp γ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$= 9 and supp $\gamma_{se}^{\times}$[G]+supp$\gamma_{se}^{\times}[G']$ = 5.

Theorem 2.10: Let G = $D_{1,s}$, s∈ N and G' = $D_{1,s[v]}$ be the graph obtained by switching the vertex v of the bistar $D_{r,s}$.Then

i. suppγ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$ = 2s + 10

ii. supp $\gamma_{se}^{\times}$[G] +supp$\gamma_{se}^{\times}[G']$ = 4(s+1)

Proof: Let G = $D_{1,s}$, s∈ N. Let V(G) = $\{u, v, u_1, u_2, \dots, u_s, v_1, v_2, \dots v_s\}$. Let G' = $D_{1,s[v]}$= $P_3 \cup sK_1$. Then V(G') =V(G), deg u = deg v = 1, deg u_1= 2 and deg v_i= 0, 1 ≤i≤ s. S = $\{u_1, v_i$ / 1 ≤i≤ s} is the unique γ_{se}- set of G'(see[12])

i. suppγ_{se}^{+}[G] = 2s + 6. Consider S of G'. supp$\gamma_{se}^{+}[u_1]$ = degu_1 + deg u + deg v = 4 and supp$\gamma_{se}^{+}[v_i]$ = 0, 1 ≤i≤ s. Therefore supp $\gamma_{se}^{+}[G']$ = supp $\gamma_{se}^{+}[u_1]$ + $\sum_{i=1}^{s}$ supp $\gamma_{se}^{+}[v_i]$ = 4. Hence suppγ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$ = 2s +10.

ii. supp$\gamma_{se}^{\times}$[G] = 4(s+1) and supp$\gamma_{se}^{\times}[G']$ = 0. Hence supp $\gamma_{se}^{\times}$[G]+supp $\gamma_{se}^{\times}[G']$ = 4(s+1)

Theorem 2.11: Let G = $D_{1,s}$, s ∈ N and G' = $D_{1,s[u_1]}$ be the graph obtained by switching the vertex u_1 of the bistar $D_{r,s}$.Then

i. suppγ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$ = 6s + 10

ii. supp$\gamma_{se}^{\times}$[G] + supp $\gamma_{se}^{\times}[G']$ = (s+1) [4+$2^s(s+2)$]

Proof: Let G = $D_{1,s}$, s ∈ N. Let V(G) = $\{u, v, u_1, u_2, \dots, u_s, v_1, v_2, \dots v_s\}$ Let G' = $D_{1,s[u_1]}$. Then V(G') = V(G), deg u = 1, deg v = s + 2, deg u_1 = s + 1 and deg v_i = 2, 1 ≤i≤s. S = {v} is the unique γ_{se}- set of G'[12].

i. suppγ_{se}^{+}[G] = 2s + 6. supp$\gamma_{se}^{+}[G']$ = supp $\gamma_{se}^{+}[v]$ = $\sum_{u\in N[v]} \deg u$ = degv + deg u + deg u_1 + $\sum_{i=1}^{s} \deg v_i$ = s+2+ 1 + (s + 1) + 2s = 4s + 4.

Hence suppγ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$ = 6s + 10.

iii. supp$\gamma_{se}^{\times}$[G] = 4(s+1), supp$\gamma_{se}^{\times}[G']$ = supp$\gamma_{se}^{\times}[v]$ = $\prod_{u\in N[v]}$ deg$u$ = degv×deg u × degu_1 × $\prod_{i=1}^{s} \deg v_i$ = (s+2)× (s + 1)2^s. Hence supp $\gamma_{se}^{\times}$[G]+supp $\gamma_{se}^{\times}[G']$ = 4(s+1) + (s+2) (s + 1)2^s = (s+1) [4+$2^s(s+2)$]

Theorem 2.12: Let G = $D_{r,s}$, r, s∈ N and G' = $D_{r,s[u,v]}$ be the graph obtained by switching both the central vertices u and v of the bistar $D_{r,s}$.Then

i. supp γ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$ =$(r+1)^2$ + 4s + 3r

ii. supp $\gamma_{se}^{\times}$[G] + supp $\gamma_{se}^{\times}[G']$ $= s$(r + $1)^{r+1}$ + rs

Proof: Let G = $D_{r,s}$, r, s ∈N. Let V(G) = $\{u, v, u_1, u_2, \dots, u_{sr}, v_1, v_2, \dots v_s\}$ Let G' = $D_{r,s[u,v]}$= $K_{1,r} \cup K_{1,s}$. Then V(G') =V(G). deg u = s, deg v = r, degu_i = deg v_j = 1, where 1≤i≤ r and 1 ≤ j ≤ s . $\{u, v\}$ is the unique γ_{se}- set of G'[12].

i. supp γ_{se}^{+} [G] = $(r+1)^2$ + r + 2s.supp$\gamma_{se}^{+}[G']$ = supp γ_{se}^{+}[u] + supp γ_{se}^{+}[v] = $\deg u + \deg v + \sum_{j=1}^{s} \deg v_j$ + $\sum_{i=1}^{r} \deg u_i$ = 2(s + r). Hence suppγ_{se}^{+}[G] + supp $\gamma_{se}^{+}[G']$ = $(r+1)^2$ + 4s + 3r.

ii. supp $\gamma_{se}^{\times}$ [G] = s(r + $1)^{r+1}$. supp $\gamma_{se}^{\times}[G']$ = degu $\times \deg v \times \prod_{j=1}^{s} \deg v_j$ $\times \prod_{i=1}^{r} \deg u_i$ = rs

Hence supp$\gamma_{se}^{\times}$[G] +supp $\gamma_{se}^{\times}[G']$ = s(r + $1)^{r+1}$ + rs

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REFERENCES

1. Balamurugan S, Anitha M and Karnan C (2019). Closed Support of a Graph Under Addition I, *International Journal of Mathematics Trends and Technology*, 65(5):120-122.
2. Balamurugan S, Anitha M, Karnan C and Palanikumar P (2019). Closed Support of a Graph Under Multiplication. *International Journal of Mathematics Trends and Technology*, 65(5):129-133.
3. Chartr and G, Hevia H, Jarette EB and Schultz M (1997). Subgraph distances in graphs defined by edge transfers, *Discrete Math.*, 170:63-79.
4. Gallian JA (2014). A dynamic survey of graph labeling, *The Electronics Journal of Combinatorics*, 17,DS6, 1 − 384.
5. Harary F (1969). Graph Theory, Addison – Wesley.
6. Haynes TW, Stephen Hedetniemi T and Slater PJ (1998). Fundamentals of Domination in Graphs, *Advanced Topics*, *Marcel Dekker*, Inc, New York
7. Haynes TW, Henning MA, Slater PJ and Van Der Merwe LC (2007). The complementary product of two graphs, *Bull. Inst. Combin. Appl.*, 51:21-30.
8. Meena N and Murugan K (2022). Closed Support Strong Efficient Domination number of Some Standard Graphs under Addition and *Multiplication, International*

Multidisciplinary Innovative Research Journal, VI(2):49-56.

9. Meena N, Subramanian A and Swaminathan V (2013). Graphs in which Upper Strong Efficient Domination Number Equals the Independent Number, *International Journal of Engineering and Science Invention*, 2(12):32-39
10. Meena N Subramanian A and Swaminathan V (2013). Strong efficient domination and strong independent saturation number of graphs, *International Journal of Mathematics and Soft Computing,* 3(2):41-48
11. Meena N, Subramanian A and Swaminathan V (2014). Strong Efficient Domination in Graphs, *International Journal of Innovative Science, Engineering & Technology*, 1(4):172-177.
12. Murugan K and Meena N (2016). Some Nordhaus-Gaddum Type Relations on Strong Efficient Dominating Sets, *Journal of New Results in Science*, 11: 04-16
13. Murugan and Karthikeyan (2019). Some Cycle and Star Related Nordhaus – Gaddum Type Relations on Strong Efficient Dominating Sets, Kyungpook *Mathematical Journal*, 59(3):363-375.
14. Sampathkumar E and Chikkodimath SB (1973). Semi-total graphs of a graph I.J.Karnatak *Univ. Sci.,* 18, 274-280.
15. Sastry DVSS and Raju BSP (1984). Graph equations for line graphs, total graphs, middle graphs and quasitotal graphs, *Discrete Math.*, 48:113-119.
16. Vaidya SK and Vihol PL (2011). Fibonacci and Super Fibonacci graceful labeling of some graphs, Studies in *Mathematical Sciences*, 2(2):24-35.
17. Vaidya SK and Harkar SH (2017). On Strong Domination Number of Graphs, *Applications and Applied Mathematics*, 12(1): 604-612.
18. Whitney H (1932). Congruent graphs and the connectivity graphs, *Amer. J. Math.,* 54:150-168.