International Multidisciplinary Innovative Research Journal
-An International refereed e-journal

ISSN: 2456 – 4613
Volume – VI Issue - 1
December -2021

A STUDY ON I - RECTANGULAR SEMI GROUP

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ABSTRACT

In this paper, the concept of Idempotent elements is considered as a prominent substructure in the theory of semi group. Some of the basic definitions like I – Left (Right) Singular, I - Rectangular, I - Left (Right) Regular, I – Regular, I – Left (Right) Normal, I – Normal, I – Left (Right) Quasi Normal, I – Left (Right) Semi Normal, I – Left (Right) Semi Regular Near Rings are introduced. It is proved that every I – Left Normal Semi Group is also I – Left Quasi-Normal, when S is I – Commutative. Also, a I – Left Semi Medial Semi group is I – Right Normal Semi group if it is totally Commutative. As a structure theorem, it is proved that any semi group is reduced if it is I – Rectangular and I – Mid Normal. And a Semi group has (* , IFP) if it is I – Mid Normal.

Keywords: Left (Right) Commutative Semi group, Left (Right) Semi medial Semi group, Left (Right)Singular Semi group, Normal Semi group, Rectangular Semi group, Regular Semi group, Regular Semi group, Total Semi group.

INTRODUCTION

The formal study of semigroups began in the early 20th century. The name semigroup originates in the fact that a semigroup generalizes a group by preserving only associativity and closure under the binary operation from the axioms defining a group.

The concept of a semi group is very simple and plays an important role in the development of Mathematics. The theory of semi group is similar to group theory and ring theory. The earliest major contribution to the theory of semigroups are strongly motivated by comparisons with groups and rings. In other areas of applied Mathematics, semi groups are fundamental models for linear time-invariant systems. If the semigroup operation is commutative, then the semigroup is called a *commutative semigroup* or (less often than in the analogous case of groups) it may be called anabelian semigroup.

PRELIMINARIES

In this section we present some basic concepts of semigroups and definitions needed for thestudy of this chapter.

Definition 2.1

A group S is said to be a *Semigroup* if it is closed and associative

Definition 2.2

A semigroup S is said to be

i) *Left(right) singular* if ab = a (ab = b) for all a, b ∈ S

ii) *Rectangular* if aba = a for all a, b ∈ S

iii) *Left(right) regular* if aba = ab (aba = ba) for all a, b ∈ S

iv) *Regular* if abca = abaca for all a, b, c ∈ S

v) *Total* semigroup if $s^2 = s$ for all s ∈ S

vi) *Diagonal* semigroup if $x^2 = x$; xyz = xz for all x, y ∈ S

vii) *Weakly separable* if $x^2 = xy = y^2$

Definition 2.3

A semigroup S is

i) *Left (right) normal* if abc = acb (abc = bac) for all a, b, c ∈ S

ii) *Normal* semigroup if abca = acba for all a, b, c ∈ S

iii) *Left (right) quasi normal* if abc = acbc (abc = abac) for all a, b, c ∈ S

iv) *Left (right) semi normal* if abca = acbca (abca = abcba) for all a, b, c ∈ S

v) *Left (right) semi regular* if abca = abacabca (abca = abcabaca) for all a, b, c ∈ S

Definition 2.4

A semigroup S is said to be

i) *I-left(right) Commutative* if abc = bac (cab = cba) where x, y, z are idempotents

ii) *I-Commutative* if ab = ba where a, b are idempotents

iii) *I-Medial* if xyzu = xzyu where y, z are idempotents

iv) *I-left(right) semi medial* if x^2yz = xyxz (zyx^2 = zxyx) where x, y are idempotents

v) *I-semi medial* if S is both I-left semi medial and I-right semi medial

vi) *I-left(right) distributive* if xyz = xyxz (zyx = zxyx) where x, y are idempotents

vii) *I-distributive* if S is both I-left distributive and I-right distributive

Definition 2.5

A semi group S is said to be reduced if it has no non-zero nilpotent elements.

I-RECTANGULAR SEMIGROUPS

Definitions

Definition 3.1.1

A Semi group S is said to be

i) *I-Left (Right) Singular Semi group*, if ab = a (ab = b) where b is an idempotent element.

ii) *I-Rectangular* if aba = a where b is an idempotent element.

iii) *I-Left (Right) Regular* if aba = ab (aba = ba) where a&b are idempotent elements.

iv) *I-Regular* if abca = abaca where a, b & c are idempotent elements.

v) *I-Left (Right) Normal* if abc = acb(abc = bac) where a & b are idempotent elements.

Definition 3.1.2

A Semi group S is

i) *I-Mid Normal* if abc = cba where a & c are idempotent elements.

ii) *I-Normal* abca = acba where a, b & c are idempotent elements.

iii) *I-Left (Right) Quasi Normal* if abc = acbc (abc = abac) where a, b & c are idempotentelements.

iv) *I-Left (Right) Semi Normal* if abca = acbca (abca = abcba) where b & c are idempotentelements.

v) *I-Left (Right) Semi Regular* if abca = abacabca (abca = abcabaca) where a, b & c are idempotent elements.

Definition 3.1.3

A semi group S is said to be

i) *m-power left normal* if it satisfies the identity $ab^mc^m = ac^mb^m$

ii) *m-power left quasi normal* if it satisfies the identity $ab^mc^m = ac^mb^mc^m$

iii) *m-power regular* if it satisfies the identity $ab^mc^ma = ab^mac^ma$

iv) *m-power left semi regular* if it satisfies the identity $ab^mc^ma = ab^mac^mab^mc^ma$

v) *m-power left semi normal* if $ab^mc^ma = ac^mb^mc^ma$

MAIN RESULTS

Theorem 3.2.1

Let S be I-Commutative Semi group. S is an I-Left Quasi Normal when S is I-Left Normal and the converse holds if S is Commutative.

Proof:

Let S be an I-left quasi normal semigroup

We have to prove that abc=acbc where b and c are idempotents

Since abc = acb

⇒ abcc = acbc

⇒ abc = acbc

Hence, S is I-Left Quasi Normal.

Conversely, let S be I-Left Quasi Normal & S is Commutative

We have to prove that S is I-Left Normal

(ie) to prove, abc = acb where b & c are idempotents

Since abc = acbc

⇒ abc = accb

$= ac^2b$

=acb

Hence proved.

Theorem 3.2.2:

Every idempotent element is Central if S is I-Mid Normal.

Proof:

Let E be the set of all Idempotents in a semi group S

Let e be an idempotent element in S

Let $a \in S$ and $e \in E$

We have to prove that ae=ea

$ae = ae^2$

$= e^2a$

$= ea$ for all $a \in S$

$\Rightarrow$e is a central element.

$\Rightarrow$Thus, E is Central.

Hence the proof.

Theorem 3.2.3

A total I-Left Semi Medial Semi group is I-Right Normal if it is Commutative

Proof:

Let S be a Total I-Left Semi medial Semi Group

We have to prove that S is I-Right Normal

$xyz = x^2yz$

$= (xy)(xz)$

$= yx^2z$

$= yxz$

Thus, S is I-Right Normal.

Hence the proof.

Theorem 3.2.4:

An I-Rectangular & I-Mid Normal Semi group S is reduced.

Proof:

Let S be I-Rectangular & I-Mid Normal Semi group

We have to prove that S is Reduced

S is I-Rectangular for all $a \in S$ then there exists $b \in S$ such that a=aba where b is an idempotent element

Now, $(ab)^2 = (aba)b$

$= ab \in E$

[By Theorem 3.2.2, Every idempotent element is Central if S is I-Mid Normal]

Now, $a = aba$

$= a(ab)$

$= a^2b$

Therefore, If $a^2 = 0$, then, $a = 0.\ b$

$= 0$

$a^2 = 0 \Rightarrow a = 0$

$\Rightarrow$ S has no nilpotent element.

Thus, S is Reduced.

Theorem 3.2.5:

A I-Mid Normal Semi group S has (* , IFP)

Proof:

Let S be I-Mid Normal

We have to prove that $ab = 0 \Rightarrow anb = 0$

Let $ab = 0$ for all $a, b \in S$

Then, $(ba)^2 = b(ab)a$

$= b0a$

$= 0$

By Theorem 3.2.4, S is Reduced

$\Rightarrow ba = 0$Also,

$(anb)^2 = an(ba)nb$

= an0nb

= 0

By Theorem 3.2.4, $(anb)^2 = 0$

S is I-Rectangular,

⇒ anb = 0

Thus S has (* , IFP)

Theorem 3.2.6:

A I-Left Semi Normal and Commutative semigroup S is I-Right Semi Normal.

Proof:

Let S be I-Left Semi Normal

We have to prove that S is I-Right Semi Regular

Since, abca = acbca

abca = a(bc)(ca)

= abc(ac)

= abbcaac

= abcbaca

= abcabaca

⇒ S is I-Right Semi Regular.

This completes the proof.

Theorem 3.2.7:

An I-Rectangular m-power commutative semigroup S is left (right) normal if and only if it isleft (right) quasi normal

Proof:

Let (S, .) be an I-rectangular m-power commutative semi group

Given:

(S, .) is left (right) normal we have to prove that, (S, .) as left (right) quasi normal.

Now, $ab^mc^m = ac^mb^m$ (since s is left normal)

$ab^mc^mc^m = ac^mb^mc^mab^mc^m$

$= ac^mb^mc^m$

⇒ (S, .) is left quasi normal

Conversely, let (S, .) be left quasi normal.

Then, $ab^mc^m = ac^mb^mc^m$

$= ac^mc^mb^m$

$= ac^mb^m$

⇒ (S, .) is left normal.

Hence the proof completes.

Theorem 3.2.8:

An I-Rectangular m-power commutative semi group S is left quasi normal iff it is left semiregular.

Proof:

Given that, S is left quasi normal

Therefore, $a^mc^ma = ab^mc^mc^ma$

$= aac^mb^mc^mb^ma$

$= ab^mac^mb^mc^ma$

$= ab^maac^mb^mc^ma$

$=ab^mac^mab^mc^ma$

Therefore, (S, .) is left semi regular.

Conversely, Given S is left semi regular

That is, abca = abacabca

= aabcabca

= abacabca

=abcaabca

$ab^mc^m = ab^mc^mab^mc^ma$

$= ab^mc^mb^mac^ma$

$= ab^mb^mc^mc^maa$

$= ab^mc^mc^ma$

$= ac^mb^mc^ma$

$= ac^mb^mc^m$

Therefore, (S, .) is left quasi normal.

Theorem 3.2.9:

An I-Rectangular m-power commutative semi group is left quasi normal then it is both right semi normal and right semi regular.

Proof:

Given, (S, .) is left quasi normal

That is, $ab^mc^m = ac^mb^mc^m$

$\Rightarrow ab^mc^ma = ac^mb^mc^ma$

$= ab^mc^mc^ma$

$= ab^mc^ma$

$= ab^mb^mc^ma$

$= ab^mc^mb^ma$

$\Rightarrow$ (S, .) is right semi normal

Also, $ab^mc^m = ac^mb^mc^m$

$\Rightarrow \quad ab^mc^ma = ac^mb^mc^m$

$= aac^mb^mb^mc^ma$

$= aab^mc^mb^mc^maa$

$= ab^mc^mab^mac^ma$

$\Rightarrow$ (S,.) is right semi – regular I-Rectangular Semi group.

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