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GEOMETRIC MEAN CORDIAL LABELING OF m – SUBDIVISION OF GRAPHS

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ABSTRACT

In this paper, we introduce the a concept of *Geometric Mean Cordial Labeling of* m *– Subdivision of Graphs* which is a kind of cordial labeling [2]. We construct m – subdivision of graphs for standard graphs such as path and cycle and expand path and cycle by applying the operation subdivision [2.2]. Also we check whether the m-subdivision of graphs are geometric mean cordial graphs or not.

Keywords: Geometric mean cordial labeling, geometric mean cordial graph, path, cycle, subdivision, subdivisional vertices.

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1. INTRODUCTION

Today, Graph labeling [3] especially cordial labeling [2] plays an important role in the study of Graph Theory [1] in Mathematics. Many problems in network communication [1] use this cordial labeling for data organization, computational devices and for the flow of computation. Cordial labeling [2] was first introduced by Cahit in the year 1987. Using the concept of geometric mean cordial labeling, we investigate whether m – subdivision of graphs admit geometric mean cordial labeling or not. The m – subdivision of graphs give a long expansion and growth to the connected graphs [4] such as path, cycle etc.

2. *m* – Subdivision of Graphs

Definition 2.1. [4 , 6] A ***subdivision*** of an edge *e* of a graph *G* is the subdivision of edge by introducing new vertices.

Definition 2.2. [1] A ***subdivision*** of a graph *G* denoted by $S(G)$ (known as sometimes expansion) is a graph resulting from the subdivision of edges in *G*. The subdivision of some edge with the end points *u* and *v* yields a graph containing one new vertex *w*, and with the edge set replacing *e* by two edges *uw* and *wv* .

Definition 2.3. The operation $S_m(G)$ of a graph *G* is a graph *G* resulting from the subdivision of edges by *m* vertices in *G* .

For $m = 1$, $S_1(G) = S(G)$ *where* $S(G)$ *denotes subdivision of* G.

For $m \geq 2$, $S_m(G) = S(S_{m-1}(G))$.

Definition 2.4. [6] Let $G = (V, E)$ be a graph and *f* be a mapping from $V(G) \rightarrow \{0, 1, 2\}$. For each edge *uv*, assign the label $\lceil \sqrt{f(u)f(v)} \rceil$, *f* is called a ***geometric mean cordial labeling*** if $|v_f(i) - v_f(j)| \leq 1$ *and* $|e_f(i) - e_f(j)| \leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with *x*, $x \in \{0, 1, 2\}$ respectively. A graph with a geometric mean cordial labeling is called ***geometric mean cordial graph.***

Result 2.5. The subdivision of the graph P_n is $S(P_n) \cong P_{2n-1}$ where P_{2n-1} is a path of $2n - 1$ vertices and $2n - 2$ edges.

Result 2.6. The subdivision of the graph C_n is $S(C_n) \cong C_{2n}$ where C_{2n} is a cycle of $2n$ vertices and $2n$ edges.

3. *m* - subdivision of standard graphs.

Path.

From the Result 2.5, it follows that $S(P_n) \cong P_{2n-1}$ where P_{2n-1} is a path of $2n - 1$ vertices and $2n - 2$ edges.

Now $S_1(P_{m+1}) = S(P_{m+1}) = P_{2(m+1)-1} = P_{2m+1}$

$$S_2(P_{m+1}) = S(S_1(P_{m+1})) = S(P_{2m+1}) = P_{2(2m+1)-1} = P_{4m+1}$$

$$S_3(P_{m+1}) = S(S_2(P_{m+1})) = S(P_{4m+1}) = P_{2(4m+1)-1} = P_{8m+1}$$

In general, we have

$$S_m(P_{m+1}) = S(S_{m-1}(P_{m+1})) = S(P_{(2^{m-1}.m)+1}) = P_{2(2^{m-1}.m+1)-1} = P_{(2^m.m)+1}$$

Then $S_m(P_{m+1})$ is a path of $(2^m . m) + 1$ vertices and $2^m . m$ edges.

Cycle.

From the Result 2.6, it follows that $S(C_n) = C_{2n}$ where C_{2n} is a cycle of $2n$ vertices and $2n$ edges.

Now $S_1(C_n) = S(C_n) = C_{2n}$

$S_2(C_n) = S(S_1(C_n)) = S(C_{2n}) = C_{4n}$

$S_3(C_n) = S(S_2(C_n)) = S(C_{4n}) = C_{8n}$

$S_{m-1}(C_n) = S(S_{m-2}(C_n)) = S(C_{2^{m-2}.n}) = C_{2.2^{m-2}.n} = C_{2^{m-1}.n}$

In general, we have, $S_m(C_n) = C_{2^m.n}$

Then $S_m(C_n)$ is a cycle of $2^m . n$ vertices and $2^m . n$ edges.

4. Geometric mean cordial labeling of *m* – subdivision of graphs.

The following results will be used to find geometric mean cordial labeling of *m*- subdivision of graphs.

Theorem 4.1. [5] *The path P_n is geometric mean cordial.*

Theorem 4.2. [5] *The cycle C_n is geometric mean cordial when $n \equiv 1, 2 \ (mod 3)$.*

Theorem 4.3. *$S(P_n)$ is geometric mean cordial.*

Proof: Let $P_n : u_1, u_2, \dots, u_n$ be the path of n vertices and $n-1$ edges. We subdivide the $n-1$ edges of P_n. Now we get $n-1$ subdivisional vertices. Let $s_1, s_2, \dots, s_{n-1}$ be the subdivisional vertices of P_n. From the Result 2.5, it follows that $S(P_n) \cong P_{2n-1}$ where P_{2n-1} is a path of $2n-1$ vertices and $2n-2$ edges. From the Theorem 4.1 [5], it follows that $S(P_1) \cong P_1$ and $S(P_2) \cong P_3$ which are geometric mean cordial. This Theorem is dealt according to 3 cases by using congruence modulo n.

Case (i) : $n \equiv 0 \ (mod 3)$. Let $n = 3t, t \geq 1$

Now the path P_{2n-1} has $6t - 1$ vertices and $6t - 2$ edges. Let $V(P_{2n-1}) = V_1 \cup V_2$ *where* $V_1 = \{u_1, u_2, \dots, u_n\}$ and $V_2 = \{s_1, s_2, \dots, s_{n-1}\}$

Define the function $f : V_1 \rightarrow \{0, 1, 2\}$ for $3t$ vertices of P_n by

$f(u_i) = 2, \quad 1 \leq i \leq t,$

$f(u_{i+t}) = 1, \quad 1 \leq i \leq t,$

$f(u_{i+2t}) = 0, \quad 1 \leq i \leq t.$

Consider vertices of V_2 If $t = 1$, then there exists 2 sub divisional vertices. The possible labeling of these two sub divisional vertices namely s_1 and s_2 are 1 and 0, or 1 and 2 or 2 and 1. In these three combinations, we get geometric mean cordial.

If $t > 1$, $3t - 1$ subdivisional vertices are labeled according to the following function.

$f(s_i)$ $= 2,\quad 1 \le i \le t-1,$

$= 1,\quad t \le i \le 2t-1,$

$= 0,\quad 2t \le i \le 3t-1.$

Then $v_f(0) = 2t,\ v_f(1) = 2t,\ v_f(2) = 2t - 1,$

$e_f(0) = 2t,\ e_f(1) = 2t - 1,\ e_f(2) = 2t - 1.$

Case (ii) : $n \equiv 1(mod3)$. Let $n = 3t + 1, t \ge 1$

Now the path P_{2n-1} has $6t + 1$ vertices and $6t$ edges. Let $V(P_{2n-1}) = V_1 \cup V_2$ where $V_1 = \{u_1, u_2, \ldots, u_n\}$ and $V_2 = \{s_1, s_2, \ldots, s_{n-1}\}$

Define the function $f : V_1 \to \{0, 1, 2\}$ for $3t + 1$ vertices of P_n by

$f(u_i)$ $= 2,\quad 1 \le i \le t,$

$f(u_{i+t})$ $= 1,\quad 1 \le i \le t+1,$

$f(u_{1+i+2t}) = 0,\quad 1 \le i \le t.$

Consider vertices of V_2

$f(s_i)$ $= 2,\quad 1 \le i \le t,$

$= 1,\quad t+1 \le i \le 2t,$

$= 0,\quad 2t+1 \le i \le 3t.$

Then $v_f(0) = 2t,\ v_f(1) = 2t + 1,\ v_f(2) = 2t,$

$e_f(0) = 2t,\ e_f(1) = 2t,\ e_f(2) = 2t.$

Case (iii): $n \equiv 2(mod3)$. Let $n = 3t + 2$.

Now the path P_{2n-1} has $6t + 3$ vertices and $6t + 2$ edges. Let $V(P_{2n-1}) = V_1 \cup V_2$ where $V_1 = \{u_1, u_2, \ldots, u_n\}$ and $V_2 = \{s_1, s_2, \ldots, s_{n-1}\}$

Define the function $f : V_1 \to \{0, 1, 2\}$ for $3t + 2$ vertices of P_n by

$f(u_i)$ $= 2,\quad 1 \le i \le t,$

$f(u_{i+t})$ $= 1,\quad 1 \le i \le t+1,$

$f(u_{1+i+2t}) = 0,\quad 1 \le i \le t+1.$

Consider vertices of V_2

$f(s_i)$ $= 2,\quad 1 \le i \le t,$

$= 1,\quad t+1 \le i \le 2t,$

$= 0,\quad 2t+1 \le i \le 3t+1.$

Then $v_f(0) = 2t + 2,\ v_f(1) = 2t + 1,\ v_f(2) = 2t,$

$e_f(0) = 2t + 2,\ e_f(1) = 2t,\ e_f(2) = 2t.$

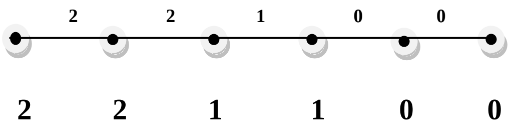
The labeling defined does not satisfy the vertex and edge condition. To make it into a geometric mean cordial labeling, we change the vertex labeled 0 which is adjacent to 1 by the labeling 2. We get $v_f(0) = 2t + 1,\ v_f(1) = 2t + 1,\ v_f(2) = 2t + 1.$

In this case, we get $e_f(0) = 2t + 1\ \ e_f(1) = 2t,\ e_f(2) = 2t + 1.$

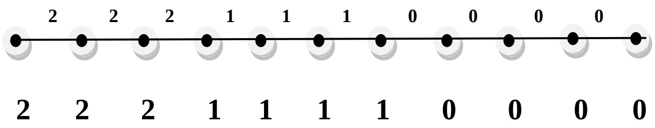
Now it satisfies both the vertex and edge condition.

In all the three cases, we see that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$, f is a geometric mean cordial labeling and hence the subdivision of a graph $S(P_n)$ is geometric mean cordial.

Example 4.4. Geometric mean cordial labeling of $S(P_6)$ is given below.



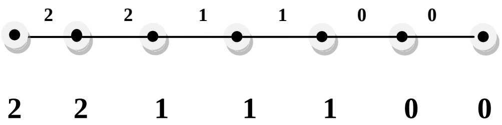
P_6



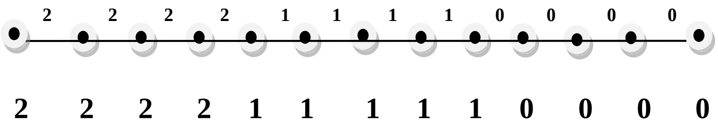
$S(P_6) \cong P_{11}$

Here, in $S(P_6)$, $v_f(0) = 4$, $v_f(1) = 4$, $v_f(2) = 3$ and $e_f(0) = 4$, $e_f(1) = 3$, $e_f(2) = 3$.

Example 4.5. Geometric mean cordial labeling of $S(P_7)$ is given below.



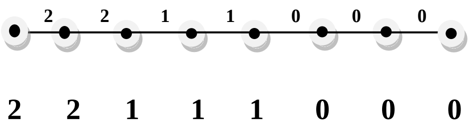
P_7



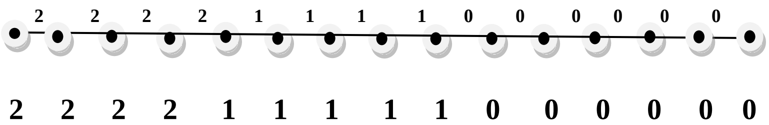
$S(P_7) \cong P_{13}$

Here, in $S(P_7)$, $v_f(0) = 4$, $v_f(1) = 5$, $v_f(2) = 4$ and $e_f(0) = e_f(1) = e_f(2) = 4$..

Example 4.6. Geometric mean cordial labeling of $S(P_8)$ is given below.



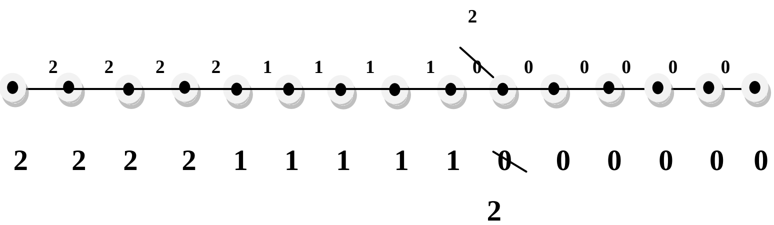
P_8



$S(P_8) \cong P_{15}$

Here, in $S(P_8)$, $v_f(0) = 6$, $v_f(1) = 5$, $v_f(2) = 4$ and $e_f(0) = 6$, $e_f(1) = 4$, $e_f(2) = 4$.

We see that the above labeling is not geometric mean cordial labeling. To make the geometric mean cordiality, we change the label as follows.



Now $v_f(0) = 5$, $v_f(1) = 5$, $v_f(2) = 5$ and $e_f(0) = 5$, $e_f(1) = 4$, $e_f(2) = 5$.

Theorem 4.7. *$S_m(P_{m+1})$ is geometric mean cordial.*

Proof. We know that $S_m(P_{m+1})$ is a graph of $2^m . m + 1$ vertices and $2^m. m$ edges. The theorem is easily verified for $m = 0, 1, 2$. If $m = 0$, we get a graph P_1 and is of 1 vertex and no edge. Now the graph has no subdivision. If $m = 1$, we get subdivision of a graph $S_1(P_2) = S(P_2) \cong P_3$, From the Theorem 4.1 [5], it follows P_3 is geometric mean cordial, $S_1(P_2)$ is geometric mean cordial. If $m = 2$, we get a subdivision of a graph $S_2(P_3) = S(S_1(P_3)) = S(P_3) \cong P_5$. From the Theorem 4.1 [5], it follows P_5 is geometric mean cordial, $S_2(P_3)$ is geometric mean cordial and hence $S(P_n)$ is geometric mean cordial.

Case (i) : $m \equiv 0 \ (mod 3)$. Let $m = 3t$, $t \geq 1$.

Now the path has $(2^{3t} \cdot 3t) + 1$ vertices and $2^{3t} \cdot 3t$ edges. The labeling is as follows. If we assign $0^{'s}$ to $2^{3t} \cdot t$ vertices, $1^{'s}$ to $2^{3t} \cdot t + 1$ vertices and $2^{'s}$ to $2^{3t} \cdot t$ vertices, then

$v_f(0) = 2^{3t} \cdot t$, $v_f(1) = 2^{3t} \cdot t + 1$, $v_f(2) = 2^{3t} \cdot t$ and

$e_f(0) = 2^{3t} \cdot t$, $e_f(1) = 2^{3t} \cdot t$, $e_f(2) = 2^{3t} \cdot t$,

In this case, we see that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$, f is a geometric mean cordial labeling and hence the m – subdivision of a graph $S_m(P_{m+1})$ is a geometric mean cordial graph.

Case (ii) : $m \equiv 1 \ (mod 3)$. Let $m = 3t + 1$.

Now the path has $(2^{3t+1} \cdot (3t + 1)) + 1$ vertices and $2^{3t+1} \cdot (3t + 1)$ edges. The labeling is as follows. There are two subcases.

Subcase (i): $v_f(0) = \dfrac{(2^{3t+1}(3t+1))-1}{3}+1$, $v_f(1) = \dfrac{(2^{3t+1}(3t+1))-1}{3}+1$,

$$v_f(2) = \frac{(2^{3t+1}(3t+1))-1}{3}, \ t = 1, 3, 5, \ldots.$$

In this subcase, we get $e_f(0) = \dfrac{(2^{3t+1}(3t+1))-1}{3}+1$, $e_f(1) = \dfrac{(2^{3t+1}(3t+1))-1}{3}$,

$$e_f(2) = \frac{(2^{3t+1}(3t+1))-1}{3}.$$

Subcase (ii): $v_f(0) = \dfrac{(2^{3t+1}(3t+1))-2}{3}+1$, $v_f(1) = \dfrac{(2^{3t+1}(3t+1))-2}{3}+1$,

$$v_f(2) \ \frac{(2^{3t+1}(3t+1))-2}{3}+1, \ t = 2, 4, 6, \ldots.$$

In this subcase, we get $e_f(0) = \dfrac{(2^{3t+1}(3t+1))-2}{3}+1$, $e_f(1) = \dfrac{(2^{3t+1}(3t+1))-2}{3}$,

$$e_f(2) \ \frac{(2^{3t+1}(3t+1))-2}{3}+1.$$

In all the subcases, we see that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$, f is a geometric mean cordial labeling and hence the m – subdivision of a graph $S_m(P_{m+1})$ is a geometric mean cordial graph.

Case (iii) : $m \equiv 2 \ (mod 3)$. Let $m = 3t + 2$.

Now the path has $(2^{3t+2} \cdot 3t+2)+1$ vertices and $2^{3t+2} \cdot 3t+2$ edges. The labeling is as follows. There are two subcases.

Subcase (i): $v_f(0) = \frac{(2^{3t+2}(3t+2))-1}{3}+1$, $v_f(1) = \frac{(2^{3t+2}(3t+2))-1}{3}+1$,

$v_f(2) = \frac{(2^{3t+2}(3t+2))-1}{3}$ $t = 1, 3, 5, \ldots.$

In this subcase, we get $e_f(0) = \frac{(2^{3t+2}(3t+2))-1}{3}+1$, $e_f(1) = \frac{(2^{3t+2}(3t+2))-1}{3}$,

$e_f(2) = \frac{(2^{3t+2}(3t+2))-1}{3}$.

Subcase (ii): $v_f(0) = \frac{(2^{3t+2}(3t+2))-2}{3}+1$, $v_f(1) = \frac{(2^{3t+2}(3t+2))-2}{3}+1$,

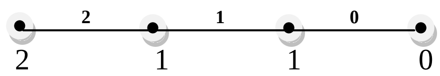
$v_f(2) = \frac{(2^{3t+1}(3t+2))-2}{3}+1$ $t = 2, 4, 6, \ldots.$

In this subcase, we get $e_f(0) = \frac{(2^{3t+2}(3t+2))-2}{3}+1$, $e_f(1) = \frac{(2^{3t+2}(3t+2))-2}{3}$,

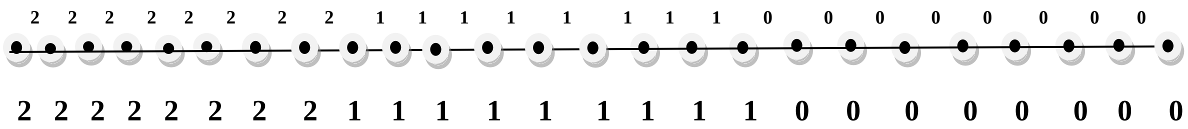
$e_f(2) = \frac{(2^{3t+2}(3t+2))-2}{3}+1$.

Now we see that $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$ for all $i, j \in \{0, 1, 2\}$, f is a geometric mean cordial labeling and hence the m – subdivision of a graph $S_m(P_{m+1})$ is a geometric mean cordial graph.

Example 4.8. Geometric mean cordial labeling of $S_3(P_4)$ is given below.



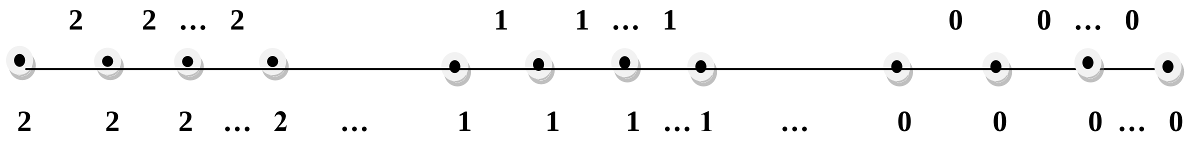
P_4



$S_3(P_4) \cong P_{25}$

Here $v_f(0) = 8$, $v_f(1) = 9$, $v_f(2) = 8$ and $e_f(0) = 8, e_f(1) = 8, e_f(2) = 8$.

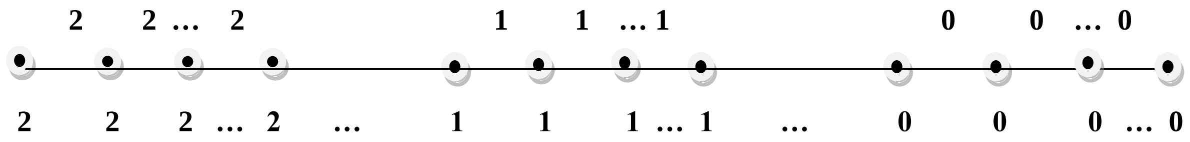
Example 4.9. Geometric mean cordial labeling of $S_4 (P_5)$ is given below.



$S_4 (P_5) \cong P_{65}$

Here $v_f (0) = 22$, $v_f (1) = 22$, $v_f (2) = 21$ and $e_f (0) = 22$, $e_f (1) = 21$, $e_f (2) = 21$.

Example 4.10. Geometric mean cordial labeling of $S_5 (P_6)$ is given below.



$S_5 (P_6) \cong P_{161}$

Here $v_f (0) = 54$, $v_f (1) = 54$, $v_f (2) = 53$ and $e_f (0) = 54$, $e_f (1) = 53$, $e_f (2) = 53$..

Theorem 4.11 . *$S (C_n)$ is geometric mean cordial iff $n \equiv 1,2 \ (mod3)$*

Proof. Let $C_n : u_1 \ u_2 \ ... \ u_n$ be the cycle of n vertices and n edges. Let $s_1, s_2, ..., s_n$ be the subdivisional vertices of C_n. From the Result 2.6, it follows that $S (C_n)$ is C_{2n} and has $2n$ vertices and $2n$ edges. This theorem is also dealt into 3 cases by using congruence modulo n.

Case (i) : $n \equiv 0 \ (mod3)$. Let $n = 3t$.

Now the cycle C_{2n} has $6t$ vertices and $6t$ edges. Here C_{2n} consists of $3t$ vertices of C_n and $3t$ subdivisional vertices in order. If $S (C_n)$ admits a geometric mean cordial labeling f, then we should have $v_f (0) = v_f (1) = v_f (2) = 2t$ and $e_f (0) = e_f (1) = e_f (2) = 2t$ -------- (1).

Consider $v_f (0) = 2t$. If we assign 0's to $2t$ number of vertices in $S(C_n)$, then we get $e_f (0) > 2t$, a contradiction to (1). Hence f is not a geometric mean cordial labeling.

Case (ii) : $n \equiv 1 \ (mod3)$. Let $n = 3t + 1$.

Now the cycle C_{2n} has $6t + 2$ vertices and $6t + 2$ edges. Here C_{2n} consists of $3t +1$ vertices of C_n and $3t + 1$ subdivisional vertices. Assign the label 1 to $t + 1$ vertices, and the labels 0 and 2 to remaining each of the t vertices in C_n and orderly we assign the same labeling to $3t + 1$ subdivisional vertices such that 0 to 1^{st} t subdivisional vertices $s_1, s_2, ..., s_t$, and 1 and 2 to remaining $s_{t+1}, s_{t+2}, ..., s_{2t+1}$ and $s_{2t+2}, s_{2t+3}, ...s_{3t+1}$ respectively.

Then we get, $v_f (0) =2t$, $v_f (1)= 2t + 2$, $v_f (2) = 2t$, $e_f (0) =2t+1$, $e_f (1)= 2t + 1$, $e_f (2) = 2t$, and now it does not satisfy vertex labeling. To make it into geometric mean cordial labeling, we change the one vertex labeled 1 adjacent to 2 by the labeling 2, then we get $v_f(0) = 2t$, $v_f(1) = 2t + 1$, $v_f(2) = 2t + 1$ and $e_f(0) = 2t + 1$, $e_f(1) = 2t$, $e_f(2) = 2t + 1$. If we change the one vertex labeled *1* adjacent to *0* by the labeling *2*, then it would not affect the previous edge labeling , it would gives the same result.

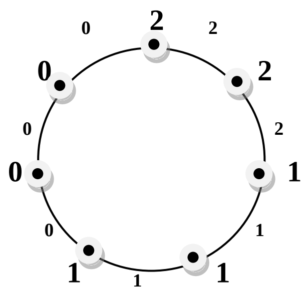
Case (iii) : $n \equiv 2 \pmod 3$. Let $n = 3t + 2$.

Now the cycle C_{2n} has $6t + 4$ vertices and $6t + 4$ edges. Here C_{2n} consists of $3t + 2$ vertices of C_n and $3t + 2$ subdivisional vertices. Assign the label *0* to *t* vertices, and the labels *1* and *2* to remaining each of the $t + 1$ vertices in C_n and orderly we assign the same labeling to $3t + 2$ subdivisional vertices such that *0* to 1^{st} *t* subdivisional vertices $s_1, s_2, \ldots, s_t$, and *1* and *2* to remaining $s_{t+1}, s_{t+2}, \ldots, s_{2t+1}$ and $s_{2t+2}, s_{2t+3}, \ldots s_{3t+2}$ respectively. Then we get, $v_f(0) = 2t$, $v_f(2) = 2t + 2$, $v_f(1) = 2t + 2$, and $e_f(0) = 2t+1$, $e_f(1) = 2t + 1$, $e_f(2) = 2t+2$, now it does not satisfy vertex labeling. To make it into cordial labeling, we change one vertex labeled *1* adjacent to a vertex labeled *0*, by the labeling *0* and one vertex labeled *2* adjacent to a vertex labeled *1* by the labeling *1*, then we get $v_f(0) = 2t + 1$, $v_f(1) = 2t + 2$, $v_f(2) = 2t + 1$. In this subcase, we get, $e_f(0) = 2t + 2$, $e_f(1) = 2t + 1$, $e_f(2) = 2t + 1$.

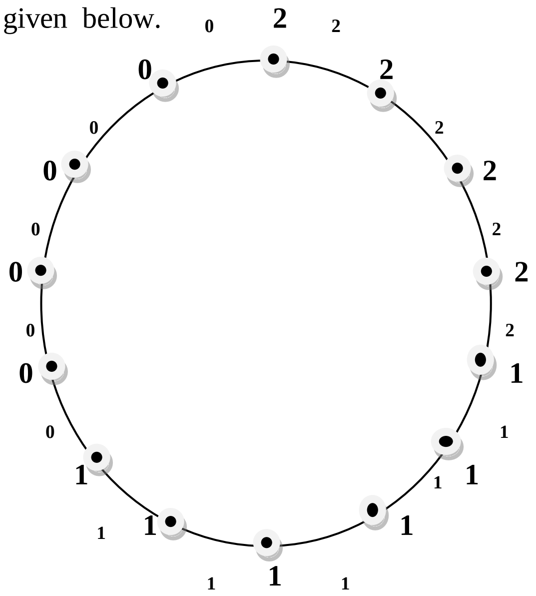
In all cases, we see that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$, *f* is a geometric mean cordial labeling and hence the subdivision of a graph $S(C_n)$ is a geometric mean cordial graph.

Example 4.12.

Geometric mean cordial labeling of $S(C_7)$ is given below.



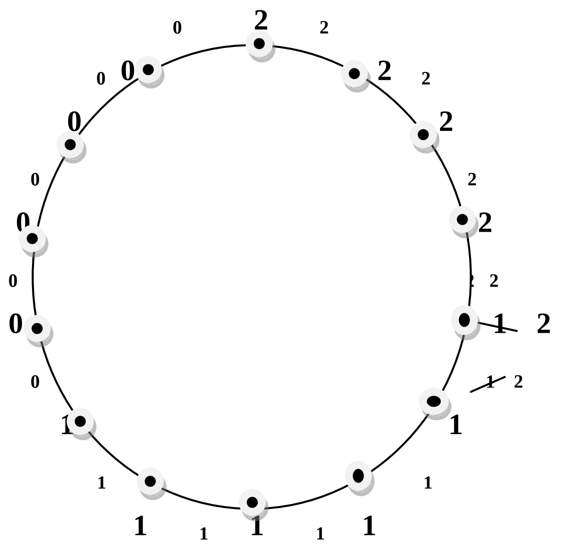
C_7



$S(C_7) \cong C_{14}$

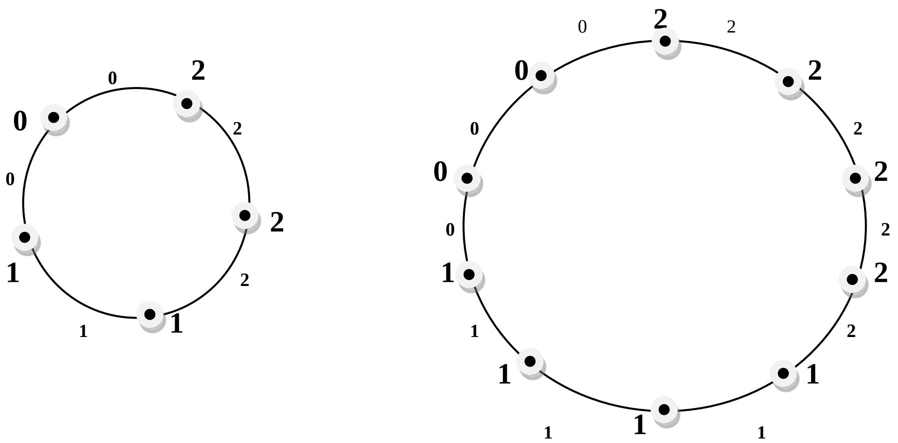
Here $v_f(0) = 4$, $v_f(1) = 6$, $v_f(2) = 4$ and $e_f(0) = 5$, $e_f(1) = 5$, $e_f(2) = 4$.

We see that the above labeling is not geometric mean cordial labeling. To make the geometric mean cordiality, we have the following changes of label as follows.



Here $v_f\ (0) = 4,\ v_f\ (1) = 5,\ v_f\ (2) = 5$ and $e_f\ (0) = 5, e_f\ (1) = 4, e_f\ (2) = 5$

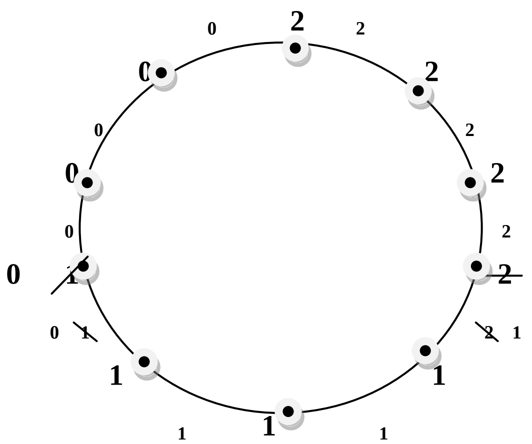
Example 4.13. Geometric mean cordial labeling of $S\ (C_5)$ is given below.



C_5 $S\ (C_5) \cong C_{10}$

Here in $S\ (C_5) = C_{10}$, $v_f\ (0) = 2$, $v_f\ (1) = 4$, $v_f\ (2) = 4$ and $e_f\ (0) = 3, e_f\ (1) = 3, e_f\ (2) = 4.$

We see that the above labeling is not geometric mean cordial labeling. To make the geometric mean cordiality, we have the following changes of label as follows.



Here $v_f\ (0) = 3,\ v_f\ (1) = 4,\ v_f\ (2) = 3$ and $e_f\ (0) = 4, e_f\ (1) = 3, e_f\ (2) = 3.$

Theorem 4.14. $S_m(C_n)$ is geometric mean cordial iff $n \equiv 1,2 \ (mod3)$

Proof. We know that $S_m(C_n)$ is the graph of $2^m . n$ vertices and $2^m . n$ edges.

Case (i) : $n \equiv 0 \ (mod3)$. Let $n = 3t$. Let $t \geq 1$.

Now the graph consists of $2^m . 3t$ vertices and $2^m . 3t$ edges. If f admits a geometric mean cordial labeling, then it should be

$v_f(0) = v_f(1) = v_f(2) = 2^m . t$ and

$e_f(0) = 2^m . t$, $e_f(1) = 2^m . t$, $e_f(2) = 2^m . t$. When we assign 0^s to $2^m . t$ vertices,

we get $e_f(0) > 2^m t$. Hence f is not geometric mean cordial labeling.

Case (ii) : $n \equiv 1 \ (mod3)$. Let $n = 3t+1$. Let $t \geq 1$.

Now the graph $S_m(C_n)$ consists of $2^m . (3t+1)$ vertices and $2^m . (3t+1)$ edges. In this case, $S_m(C_{3t+1}) \cong C_{2^m . (3t+1)}$ is a cycle that is geometric mean cordial.

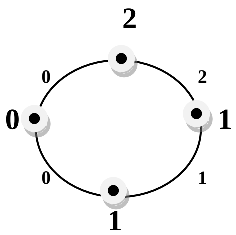
Case (iii) : $n \equiv 2 \ (mod3)$. Let $n = 3t+2$. Let $m \geq 1$.

Now the graph consists of $2^m . (3t+2)$ vertices and $2^m . (3t+2)$ edges. In this case, $S_m(C_{3t+2}) \cong C_{2^m . (3t+2)}$ is a cycle that is geometric mean cordial.

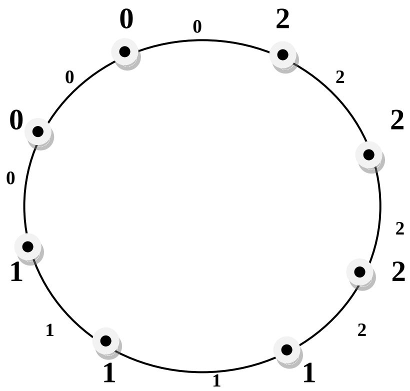
In all cases, we see that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$, f is a geometric mean cordial labeling and hence the m - subdivision of a graph $S_m(C_n)$ is a geometric mean cordial graph.

Example 4. 15

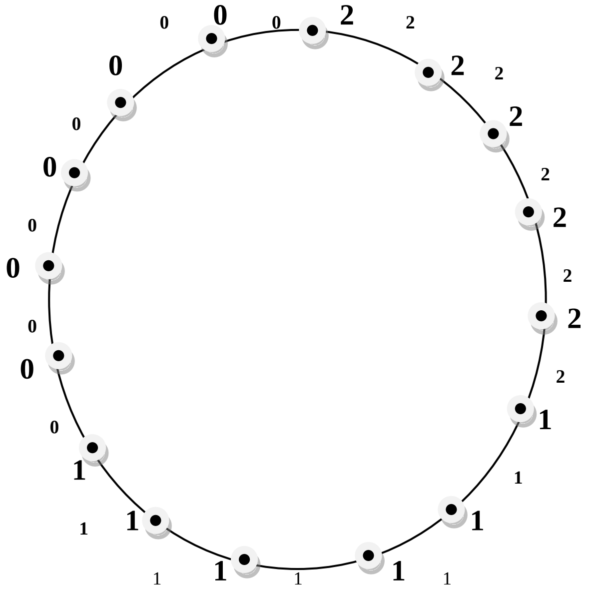
Geometric mean cordial labeling of $S_2(C_4)$ is given below



C_4



$S_1(C_4) \cong C_8$



$S_2 (C_4) \cong C_{16}$

Here in $S_2 (C_4)$, $v_f (0) = 5$, $v_f (1) = 6$, $v_f (2) = 5$ and $e_f (0) = 6$, $e_f (1) = 5$, $e_f (2) = 5$.

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