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ON COMPLEMENTS AND RELATIVE COMPLEMENTS OF SOFT ROUGH LATTICES

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ABSTRACT

Soft rough set is the generalization of rough set with respect to the soft approximation space. The complements of the soft rough sets, the complements and relative complements in soft rough lattice are given in [15, 16]. In this paper we discuss the theorem that every bounded relatively complemented soft rough lattice is complemented.

MSC: 06C20

Keywords: Soft rough set, soft rough lattice, complement of soft rough set, complements and relative complements of soft rough lattices.

1.INTRODUCTION

Soft set theory and rough set theory are both treated as the mathematical tools to deal with uncertainty. A connection between these two have been made by Feng et al.[1] who introduced the notion of soft rough set. S.K. Roy et al.[13] has found the algebraic connection between soft rough set and thereby introduced the notion of soft rough lattice in the soft approximation space. The complements can be found for both soft set as well as rough set. A soft rough lattice is a lattice structure on soft rough sets based on soft approximation space. The complements of soft rough set, complements and relative complements of soft rough lattices are discussed in [15,16]. In this paper we discuss the theorem that every bounded relatively complemented soft rough lattice is complemented and its converse.

2. Preliminaries

Definition 2.1. Let S=(F,A) be a soft set over U. Then the pair P=(U,S) is called a soft approximation space. Let X⊆ U.We define the following operations on P

$\underline{apr}(X)=\bigcup_{a\in A}\{F(a): F(a) \subseteq X\}$

$\overline{apr}(X) = \bigcup_{a\in A}\{F(a): F(a) \cap X \neq \varphi\}$

which are called soft lower and soft upper approximations respectively of X and the pair($\underline{apr}$(X),$\overline{apr}$(X)) is called soft rough set with respect to P and is denoted by $S_r(X)$. The set of all soft rough sets over U is denoted by $S_R(U)$ with respect to some soft approximation space P.

Definition 2.2. Let $S_r(X)$ = ($\underline{apr}$(X),$\overline{apr}$(X)) and $S_r(Y)$ = ($\underline{apr}$(Y),$\overline{apr}$(Y)) be two soft rough set. Then the soft rough union and the soft rough intersection of $S_r(X)$ and $S_r(Y)$ are defined by $S_r(X) \sqcup S_r(Y) = (\underline{apr}(X) \cup \underline{apr}(Y), \overline{apr}(X) \cap \overline{apr}(Y))$ and

$S_r(X) \sqcap S_r(Y) = (\underline{apr}(X) \cap \underline{apr}(Y), \overline{apr}(X) \cap \overline{apr}(Y))$ respectively,

where⊔ and ⊓ stand for soft rough union and intersection respectively.

Definition 2.3. Let $S_r(X)$ = ($\underline{apr}$(X),$\overline{apr}$(X)) and $S_r(Y)$ = ($\underline{apr}$(Y),$\overline{apr}$(Y)) be two soft rough set. Then $S_r(Y)$ is said to be soft rough subset of $S_r(X)$, denoted by $S_r(X) \sqsubseteq S_r(Y)$ if $\underline{apr}(X) \subseteq \underline{apr}(Y)$ and $\overline{apr}(X) \subseteq \overline{apr}(Y)$, where ⊑ stands for soft rough inclusion relation.

Definition 2.4. Let $\mathcal{L} \subseteq S_R(U)$, ∨and∧be two binary operations on $\mathcal{L}$.The algebraic structure($\mathcal{L}$, ∨, ∧ , ≼) is said to be a soft rough lattice if

(i) ∧ and ∨ are associative.
(ii) ∧ and ∨ are commutative.
(iii)∧ and ∨ satisfy absorption law.

Definition 2.5. Let ($\mathcal{L}$, ∨, ∧, ≼) be a soft rough lattice and $\mathcal{K} \subseteq \mathcal{L}$. Then ($\mathcal{K}$, ∨,∧,≼) is said to be soft rough sublattice of ($\mathcal{L}$, ∨, ∧, ≼) if and only if is closed under both operations. (i.e) If $S_r(X)$, $S_r(Y) \in \mathcal{K}$ then $S_r(X) \wedge S_r(Y) \in \mathcal{K}$ and $S_r(X) \vee S_r(Y) \in \mathcal{K}$.

Definition 2.6. If $S_r(X)$ = ($\underline{apr}$(X),$\overline{apr}$(X)) = (ϕ, ϕ) = $S_r(\phi)$ for all x ϵ E, then $S_r(\phi)$ is called a null soft rough set.

Definition 2.7. If $S_r(X)$ = ($\underline{apr}$(X),$\overline{apr}$(X)) = (U, U) = $S_r(U)$ for all x ϵ A, then $S_r(U)$ is called a A-universal soft rough set.

Definition 2.8. If A = E and $S_r(X)$ = ($\underline{apr}$(X),$\overline{apr}$(X)) = (U, U) = $S_r(U)$ for all x ϵ E, then $S_r(U)$ is called a universal soft rough set.

Definition 2.9. The complement of the soft rough set $S_r(X)$ over the soft approximation space (U,S) is defined by $S_r^c(X) = (U \setminus \overline{apr}(X)\ ,\ U \setminus \underline{apr}(X)) = (\overline{apr}^c(X), \underline{apr}^c(X))$ and is denoted by $S_r^c(X)$.

Definition 2.10. Let $(\mathcal{L}, \vee, \wedge, \preccurlyeq)$ be a soft rough lattice and $S_r(I)$ and $S_r(J) \in \mathcal{L}$. Suppose $S_r(I) \preccurlyeq S_r(J)$, $[S_r(I), S_r(J)] = \{ S_r(X) \in \mathcal{L} \ /\ S_r(I) \preccurlyeq S_r(X) \preccurlyeq S_r(J)\}$. Then $[S_r(I), S_r(J)]$ is said to be soft rough complemented if for every $S_r(X) \in [S_r(I), S_r(J)]$ there exists $S_r(Y) \in [S_r(I), S_r(J)]$ such that $S_r(X) \wedge S_r(Y) = S_r(I)$ and $S_r(X) \vee S_r(Y) = S_r(J)$. Here $S_r(Y)$ is said to be the soft rough relative complement of $S_r(Y)$.

Definition 2.11. A soft rough lattice $(\mathcal{L}, \vee, \wedge, \preccurlyeq)$ is said to be a bounded soft rough lattice if it has both the greatest element $S_r(U)$ and the least element $S_r(\phi)$.

Definition 2.12. Let $(\mathcal{L}, \vee, \wedge, \preccurlyeq)$ be a bounded soft rough lattice, for any $S_r(X) \in \mathcal{L}$, there exists $S_r^c(X) \in \mathcal{L}$ such that $S_r(X) \vee S_r^c(X) = S_r(U)$ and $S_r(X) \wedge S_r^c(X) = S_r(\phi)$. Then $S_r^c(X)$ is called the soft rough complement of $S_r(X)$.

Remark 2.13. The complement of the crisp set is not valid for the complement of the soft rough set in a soft rough lattice. i.e., $S_r(X) \vee S_r^c(X) = (\underline{apr}(\hat{X}), \overline{apr}(U)) \sqsubseteq S_r(U)$ and $S_r(X) \wedge S_r^c(X) = (\underline{apr}(\phi), \overline{apr}(\hat{X})) \sqsupseteq S_r(\phi)$ where $(\underline{apr}(\hat{X}))^c = \overline{apr}(\hat{X})$.

Note 2.14. A bounded soft rough lattice $(\mathcal{L}, \vee, \wedge, \preccurlyeq)$ is said to be soft rough complemented lattice if every soft rough set of $(\mathcal{L}, \vee, \wedge, \preccurlyeq)$ has atleast one soft rough complement.

3. Complements and relative complements in soft rough lattices

Theorem 3.1. Every bounded relatively complemented soft rough lattice is soft rough complemented but not conversely.

Proof. Let $(\mathcal{L}, \vee, \wedge, \preccurlyeq)$ be a bounded soft rough relatively complemented lattice with $S_r(\phi), S_r(U)$ as the least and the greatest element respectively.

So we have $[S_r(\phi), S_r(U)] = \{S_r(X) \in \mathcal{L} \ /\ S_r(\phi) \leq Sr(X) \leq S_r(U)\}$.

Now, let us take $S_r(Y) = S_r(\phi) \vee (S_r(U) \wedge S_r^c(X))$, where $S_r(Y) \in [S_r(\phi), S_r(U)]$.

Then, $S_r(X) \wedge S_r(Y) = S_r(X) \wedge (S_r(\phi) \vee (S_r(U) \wedge S_r^c(X)))$

$= S_r(X) \wedge (S_r(\phi) \vee S_r^c(X))$

$= S_r(X) \wedge S_r^c(X)$

$= S_r(\phi)$

$S_r(X) \vee S_r(Y) = S_r(X) \vee (S_r(\phi) \vee (S_r(U) \wedge S_r^c(X)))$

$= S_r(X) \vee (S_r(\phi) \vee S_r^c(X))$

$= S_r(X) \vee S_r^c(X)$

$= S_r(U)$

$S_r(Y)$ is the soft rough complement of $S_r(X)$ in $[S_r(\phi), S_r(U)]$.

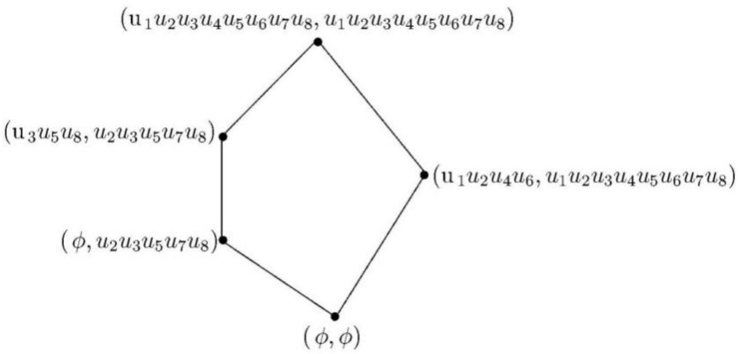
Therefore, the interval $[S_r(\phi), S_r(U)]$ is soft rough complemented.

Also every interval in $(\mathcal{L}, \vee, \wedge, \preccurlyeq)$ is soft rough complemented.

Hence $(\mathcal{L}, \vee, \wedge, \preccurlyeq)$ is a soft rough complemented lattice.

"The following counter example shows that converse of Theorem 3.1is not true".

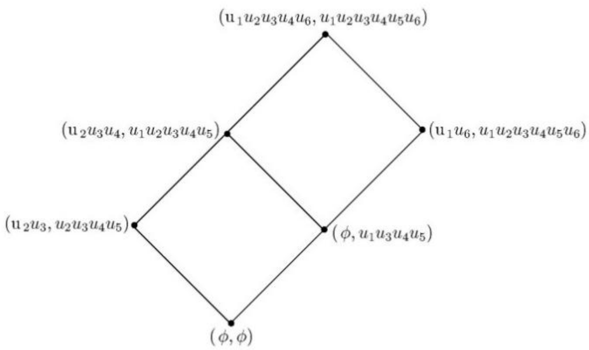
Let $U = \{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8\}$and $A = \{e_1,e_2,e_3,e_4,e_5,e_6\}$.Let S= (F,A) be a soft set over U given by $F(e_1)=\{u_1,u_2\}$, $F(e_2)=\{u_4,u_6\}$, $F(e_3)=\{u_7\}$, $F(e_4)=\{u_3,u_5,u_8\}$, $F(e_5)=\{u_1,u_4,u_6,u_7\}$, $F(e_6)=\{u_2,u_3,u_5,u_6,u_7,u_8\}$. Let $X_1= \phi$, $X_2=\{u_3,u_5\}$, $X_3=\{u_3,u_5,u_8\}$, $X_4=\{u_1,u_2,u_4,u_6\}$,$X_5=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8\}$.The soft rough sets on the soft approximation space P=(U,S) are given by $S_r(X_1) = (\phi, \phi)=S_r(\phi)$, $S_r(X_2) = (\phi,u_2u_3u_5u_7u_8)$, $S_r(X_3) = (u_3u_5u_8,u_2u_3u_5u_7u_8)$, $S_r(X_4) = (u_1u_2u_4u_6,u_1u_2u_3u_4u_5u_6u_7u_8)$, $S_r(X_5) = (u_1u_2u_3u_4u_5u_6u_7u_8,u_1u_2u_3u_4u_5u_6u_7u_8)=S_r(U)$. The Hasse diagram of the lattice $\mathcal{L} = \{S_r(\phi),S_r(X_2),S_r(X_3),S_r(X_4),S_r(U)\}$ is given by



Considering the above non modular soft rough lattice, it is a soft rough complemented lattice. The interval $[S_r(\phi), S_r(U)]$ is soft rough complemented as $S_r(X_4)$ and $S_r(X_3)$ are complement to each other and $S_r(X_2)$ and $S_r(X_4)$ are complements to each other in the soft rough interval $[S_r(\phi), S_r(U)]$.Also $S_r(\phi)$ and $S_r(U)$ are complement to each other. $S_r(X_4)$ has no soft rough complement in the soft rough interval $[S_r(\phi), S_r(X_3)]$. Therefore, the interval $[S_r(\phi), S_r(X_3)]$ is not soft rough complemented.∴ $\mathcal{L}$ is not a relatively complemented soft rough lattice.

We can clearly understand about the relatively complemented soft rough lattice by the following example.

Example 3.3. Let $U = \{u_1,u_2,u_3,u_4,u_5,u_6\}$, $A = \{e_1,e_2,e_3,e_4,e_5,e_6\}$.Let S= (F,A) be a soft set over U given by $F(e_1)=\{u_2,u_3\}$, $F(e_2)=\{u_3,u_4\}$, $F(e_3)=\{u_1,u_4\}$, $F(e_4)=\{u_3,u_5\}$, $F(e_5)=\{u_1,u_4,u_5\}$, $F(e_6)=\{u_1,u_6\}$. Let $X_1= \phi$, $X_2=\{u_2,u_3\}$, $X_3=\{u_4\}$, $X_4=\{u_3,u_4\}$,$X_5=\{u_1,u_3,u_6\}$, $X_6=\{u_1,u_2,u_3,u_4,u_6\}$.The soft rough sets on the soft approximation space P = (U,S) are given by $S_r(X_1) = (\phi, \phi)$, $S_r(X_2) = (u_2u_3,u_2u_3u_4u_5)$, $S_r(X_3) = (\phi,u_1u_3u_4u_5)$, $S_r(X_4) = (u_3u_4,u_1u_2u_3u_4u_5)$, $S_r(X_5) = (u_1u_6,u_1u_2u_3u_4u_5u_6)$, $S_r(X_6) = (u_1u_2u_3u_4u_6,u_1u_2u_3u_4u_5u_6)$. The Hasse diagram of the lattice$\mathcal{L} = \{S_r(X_1),S_r(X_2),S_r(X_3),S_r(X_4),S_r(X_5),S_r(X_6)\}$ is given by



Relatively Complemented Soft Rough Lattice

Consider the soft rough interval $[S_r(X_1), S_r(X_4)]$, for any $S_r(X_2) \epsilon [S_r(X_1), S_r(X_4)]$we have $S_r(X_3) \epsilon [S_r(X_1), S_r(X_4)]$ such that $S_r(X_2) \vee S_r(X_3) = (u_2u_3,u_1u_2u_3u_4u_5) \sqsubseteq S_r(X_4)$ and $S_r(X_2) \wedge S_r(X_3) = (\phi,u_3u_4u_5) \sqsupseteq S_r(X_1)$ which implies that $[S_r(X_1), S_r(X_4)]$ is soft rough complemented. Now, on considering the soft rough interval $[S_r(X_3), S_r(X_6)]$, for any $S_r(X_5) \epsilon [S_r(X_3), S_r(X_6)]$we have $S_r(X_4) \epsilon [S_r(X_3), S_r(X_6)]$ such that $S_r(X_5) \vee S_r(X_4) = (u_1u_3u_4u_6,u_1u_2u_3u_4u_5u_6) \sqsubseteq S_r(X_6)$ and $S_r(X_5) \wedge S_r(X_4) = (\phi,u_1u_2u_3u_4u_5) \sqsupseteq S_r(X_3)$ which implies that $[S_r(X_3), S_r(X_5)]$ is soft rough complemented. Similarly every interval in $\mathcal{L}$ is soft rough complemented. Hence the above soft rough lattice is relatively complemented.

4. CONCLUSION

The complements can be found for soft rough sets. In this paper, we have proved the theorem that every bounded relatively complemented soft rough lattice is complemented also we have shown that its converse is not true by a counter example.

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