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LABELLED DECOMPOSITION OF GRAPHS

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ABSTRACT

In this paper graph decompositions into three types of labelled graphs, namely topogenic, edge topogenic and nullset magic graphs are defined. These types of decomposition are discussed for complete graphs, wheels and Petersen graphs.

Keywords: Topogenic graphs, edge topogenic graphs and nullset magic graphs.

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INTRODUCTION

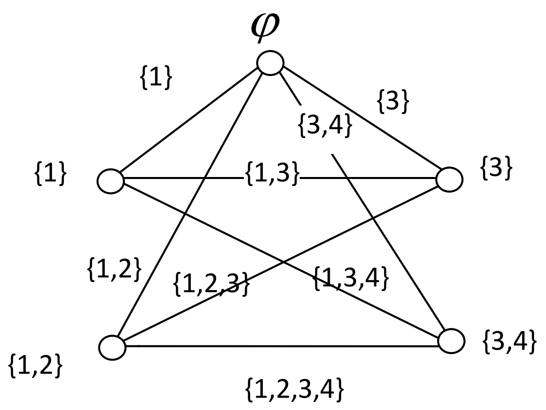
The study of graph decomposition has been one of the most important topics in graph theory and also plays an important role in the study of combinatorics of experimental designs. In this paper we have defined the decomposition of graphs into three types of labelled graphs namely, topogenic, edge topogenic and nullset magic graphs.

1.TopogenicDecomposable graphs

Definition 1.1:

A graph $G=(V, E)$ is said to be topogenic with respect to a non empty ground set X if it admits a topogenic set indexer, which is a function $f:V\rightarrow 2^X$ such that $f(V)\,U\,f^{\oplus}(E)$ is a topology on X where $f^{\oplus}:E(G)\rightarrow 2^X-\{\varphi\}$ defined by $f^{\oplus}(uv)=\big(f(u)-f(v)\big)\cup\big(f(v)-f(u)\big)$ for all $uv\in E(G)$ is the induced edge label and 2^X is the set of all subsets of a ground set X.

Example: Topogeniclabeling of graph G.



$T_f = f^{\oplus}(V) \cup f(E)$

$$= \{\phi, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{3,4\} \{1,2,3\}, \{1,3,4\}, \{1,2,3,4\}\}$$

Clearly T_f is a topology on X. Therefore G is edge topogenic

Definition1.2:

A graph G is said to be topogenic decomposable if each sub graph in the decomposition is topogenic.

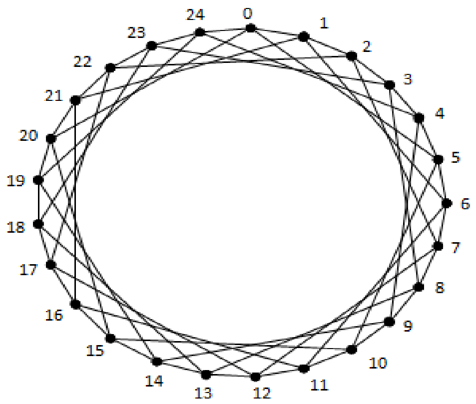
Definition1.4:

Let G be a group and $S \subseteq G$,a set of generators.Cayleygraph $\Gamma(G,S)$ is a graph,whose vertex set $V(\Gamma)$ is G and the edge set $E(\Gamma)$ consists of all pairs (g, gs) such that g∈G and s∈S.Let n≥1 be an integer.Let S be the set of divisors of n. The set $S^* = \{s, n-s : s \in S\}$ is a symmetric subset of the additive abelian group $(Z_n, \oplus_n)$ of integer modulo n. The Cayley graph of $(Z_n, \oplus)$ associated with the above symmetric subset S^* is called the divisor Cayley graph and it is denoted by $G(Z_n, S^*)$. The graph $G(Z_n, S^*)$ is the graph whose vertex set $V = \{0, 1, 2, ..., n-1\}$ and the edge set E is the set of all ordered pairs a, b such that either $a-b \in S^*$ or $b-a \in S^*$. The number of edges in G is $\frac{n|S^*|}{2}$.

It is $|S^*|$ regular.The cycle structure of these graphs has many applications in Engineering and Communication Networks.

Example:

Divisor Cayley graph $G_5 = (Z_{25},\ S^*)$ where S* = $\{1, 5, 20, 24\}$ is the generating set of G_5



Theorem1.1

The cycle $C_n,\ n \geq 3$ is topogenic.

Proof:

Let $v_1, v_2, \ldots, v_n$ be the vertices of C_n.

Let $X = \{1, 2, \ldots, n-1\}$

Label the vertices of C_n as follows:

Define $f(v_1) = \varphi$ and $f(v_i) = \{1, 2, \ldots, i-1\},\ i \in \{2, 3, \ldots, n\}$

$f^{\oplus}(v_i v_{i+1}) = \{i\}, i = 1, 2, \ldots, n-1$

$f^{\oplus}(v_n v_1) = \{1, 2, \ldots, n-1\}$

$f^{\oplus}(E) = \{\{1\}, \{2\}, \ldots, \{n-1\}, \{1, 2, \ldots, n-1\}\}$

$T_f = f(V) \cup f^{\oplus}(E)$

$=\{\varphi, \{1\}, \{2\}, ..., \{n-1\}, \{1,2\}, \{1,2,3\}, ..., \{1,2, ..., n-1\}\}$

Clearly $f(v_i) \subset f(v_{i+1})$ for $i = 2,3, ..., n-1$

$\therefore$ Finite union and finite intersection of elements in T_f will be in T_f

$\therefore T_f$ is a topology on X.

$\therefore C_n$ istopogenic.

Theorem1. 2

The divisorCayleygraph $G(Z_n, S^*)$ is topogenic decomposable when $n = p^2$,where p is a prime number.

Proof:

$G_p = (Z_{p^2}, S*)$ where S* = $\{1,\ p,\ p^2-1,\ p^2-p\}$ is the general set of G_p .

Let $\{v_0, v_2, ... v_{p^2-1}\}$ be the vertex set and

E = $\left\{ v_i v_{i+1} \middle/ 0 \le i \le p^2-1 \right\} \cup \left\{ v_i v_{i+p} \middle/ 0 \le i \le p^2-1 \right\}$ be the edge set of G_p where addition

is taken over module p^2 . G_p is decomposed into the edge disjoint cycles $C_1, C_2, C_3, ..., C_{P-1}, C$ as follows $C_1 : v_0\ v_p\ v_{2p}\ v_{3p} .... v_{p^2-p}\ v_0$

$C_2 : v_1\ v_{p+1}\ v_{2p+1}\ v_{3p+1} .... v_{p^2-p+1}\ v_1$

$C_3 : v_3\ v_{p+2}\ v_{2p+2}\ v_{3p+2} .... v_{p^2-p+2}\ v_3$

$\vdots$

$C_{p-1} : v_{p-1}\ v_{2p-1}\ v_{3p-1} .... v_{p^2-1}\ v_{p-1}$

$C : v_0\ v_1\ v_2\ v_3 .... v_{p^2-1}\ v_0$

By theorem 1.1,each cycle is topogenic .Therefore $G\left(Z_{p^2}, S^*\right)$ is topogenic decomposable.

Theorem1. 3

K_n istopogenic decomposable.

Proof:

Let $v_1, v_2, ..., v_n$ be the vertices of K_n .

For $j = 1, 2, 3, ..., n-1$, let, G_j represents the star with $V\left(G_j\right) = \left\{v_j, v_{j+1}, v_{j+2}, ..., v_n\right\}$ and $E\left(G_j\right) = \left\{v_j v_k \,/\, k = j+1, j+2, ..., n\right\}$. Then $G_j = k_{1,n-j}$, $j = 1, 2, ..., n-1$

It has been proved in $[1]$ that for each j, $k_{1,n-j}$, is topogenic.

$\therefore K_n$ istopogenic decomposable .

2.Edge topogenic decomposable graphs

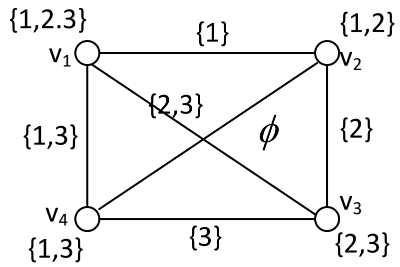
Definition 2.1:

A graph $G = (V, E)$ is said to be edge topogenic if it admits an edge topogenic set indexer $f : E \to 2^X$ such that $f(E)\, U\, f^{\oplus}(V)$ is a topology on X where

$$f^{\oplus}(v) = \left\{ \bigcup_{e_i \in E(G)} f(e_i)/e_i \text{ is incident at v} \right\} \text{and } f^{\oplus}(V) = \left\{ f^{\oplus}(v) \,/\text{v} \in \text{V} \right\}.$$

Example:

K_4 is edge topogenic.



Definition 2.2:

Agraph G is said to be edge topogenic decomposable if each sub graph in the decomposition is edge topogenic.

Definition 2.3:

The corona $G_1 \circ G_2$ of two graphs G_1 (with n_1 vertices and m_1 edges) and G_2 (with n_2 vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2 and then joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. It follows from the definition of corona that $G_1 \circ G_2$ has $n_1 + n_1 n_2$ vertices and $m_1 + n_1 m_2 + n_1 n_2$ edges.

Theorem 2.1:

The graph $C_n \circ K_1$ the corona of C_n and K_1 is edge topogenic.

Proof:

Let G = $C_n \circ K_1$ with $V(G) = \{v_1, v_2, ..., v_n\} \cup \{u_1, u_2, ..., u_n\}$ where $u_1, u_2, ..., u_n$ are the pendant vertices.

$$E(G) = \{v_1v_2, v_2v_3, ..., v_{n-1}v_n\} \cup \{v_1u_1, v_2u_2, ..., v_nu_n\}$$

Let $X = \{1, 2, ..., 2n-1\}$

Define $f(v_iu_i) = \{1, 2, ..., i-1\}$, $i = 2, 3, ..., n,$

$$f(v_nv_1) = \phi$$

$$f(v_1v_2) = \{1, 2, ..., n\}$$

$$f(v_2v_3) = \{1, 2, ..., n+1\}$$

$$\vdots$$

$$f(v_{n-2}v_{n-1}) = \{1, 2, ..., 2n-3\}$$

$$f(v_{n-1}v_n) = \{1, 2, ..., 2n-4, 2n-2\}$$

$$T_f = f(E) \cup f^{\oplus}(V) = \{\phi, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, ..., \{1,2, ..., 2n-3\},$$

$$\{1,2, ..., 2n-4, 2n-2\}\}$$

T_f is a topology on X.

$\therefore$ The graph G is edge topogenic.

Theorem 2.2:

Petersen graph P(n,1) is edge topogenic decomposable.

Proof:

Let $G = P(n,1)$.

$V(G) = \{v_1, v_2, ..., v_n\} \cup \{u_1, u_2, ..., u_n\}$ and

$E(G) = \{v_1u_1, v_2u_2, ..., v_nu_n\} \cup \{v_1v_2, v_2v_3, ..., v_{n-1}v_n\}$

Now decompose G as

(i) $G_1 = C_n \circ K_1$

$V(G_1) = \{v_1, v_2, ..., v_n\} \cup \{u_1, u_2, ..., u_n\}$

$E(G_1) = \{v_1u_1, v_2u_2, ..., v_nu_n\} \cup \{v_1v_2, v_2v_3, ..., v_{n-1}v_n\}$ and

(ii) G_2 a cycle with

$V(G_2) = \{u_1, u_2, ..., u_n\}$

$E(G_2) = \{u_1u_2, u_2u_3, ..., u_{n-1}u_n\}$

By theorem 2.1, G_1 is edge topogenic. It has been proved that in [2] that G_2 is edge topogenic.

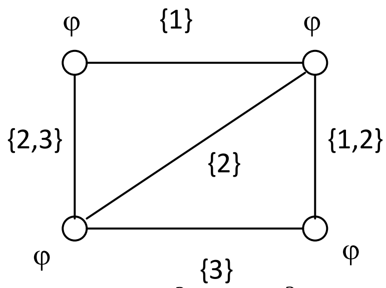
$\therefore P(n,1)$ is edge topogenic decomposable.

3.Null set magic decomposable graphs

Definition 3.1:

A graph G is said to be nullset magic if its edges can be assigned distinct subsets of a non empty set X with $2^{|X|-1} \le |E(G)| \le 2^{|X|} - 1$ such that for every vertex u of G intersection of the subsets assigned to the edges incident at u is φ. Such a set assignment to the edges of G is called a nullset magic labelling of G.

Null set magic labeling of G:



$|E(G)| = 5, \quad 2^2 \le 5 \le 2^3 - 1$

Edges of G should be assigned members of 2^X where $|x| = 3$. Let x = {1,2,3}

Intersection of the subsets assigned to the edges incident at each vertex is φ. Therefore Gis null set magic.

Definition 3.2:

A graph G is said to be nullset magic decomposable if each sub graph in the decomposition is nullset magic.

Definition 3.3:

When k copies of C_n share a common edge, it will from an n-gon book of k pages and is denoted by $B(n,k)$. The common edge is called the spine or base of the book.

Theorem 3.1

Wheel W_n, $n \ge 4$ is nullset magic.

Proof:

Let $V = \{u, v_1, v_2, ..., v_n\}$ be the vertex set where u is the centre vertex and $E = \{uv_1, uv_2, ..., uv_n\} \cup \{v_1v_2, v_2v_3, ..., v_{n-1}v_n\}$ be the edge set. For nullset magic labelling $2^{|X|-1} \le |E(G)| \le 2^{|X|} - 1$

Let $|X| = k$

Now label the edges $uv_1, uv_2, ..., uv_n$ as follows:

$$l\left(uv_{\frac{i(i-1)}{2}+1}\right) = \{i\},\ i = 1, 2, ..., k$$

$$l\left(uv_{\frac{i(i-1)}{2}+2}\right) = \{1, i\},\ i = 2, 3, ..., k$$

$$l\left(uv_{\frac{i(i-1)}{2}+3}\right) = \{2, i\},\ i = 3, 4, ..., k$$

$$l\left(uv_{\frac{i(i-1)}{2}+4}\right) = \{3, i\},\ i = 4, 5, ..., k$$

$\vdots$

$$l\left(uv_{\frac{i(i+1)}{2}}\right) = \{i-1, i\},\ i = k$$

Now, label the edges $\{v_1v_2, v_2v_3, ..., v_{n-1}v_n\}$ as follows:

$$l\left(v_i v_{\frac{i+1}{2}}\right) = \left[l\left(uv_i\right)\right]^c,\ i = 1, 2, ..., n-1$$

$$l\left(v_n v_1\right) = \left[l\left(uv_n\right)\right]^c$$

Clearly $l\left(u\right) = \varphi, l\left(v_i\right) = \varphi,$ for all $i = 1, 2, ..., n$

$\therefore W_n$ is nullset magic.

Theorem 3.2:

Every n-gonal book with odd number of pages is nullset magic decomposable for n=9

Proof:

Let G be annanogonal book graph consisting of odd number of pages, say $2m+1$

Let $V(G)=\{v_1,v_9\}\cup\{v_{ij}/2\le i\le 8,\ 1\le j\le 2m+1\}$

$$E(G)=\{v_1v_9\}\cup\{v_1v_{2j},1\le j\le 2m+1\}\cup\{v_{ij}\,v_{(i+1)j}/2\le i\le 7,$$

$$1\le j\le 2m+1\}\cup\{v_{8j}v_9,1\le j\le 2m+1\}$$

G is decomposed into $m+1$cycles, one cycle of length 9 and m cycles of length 16.

(i) Cycle of length 9:

$$v_1v_{21}v_{31}v_{41}v_{51}v_{61}v_{71}v_{81}v_9v_1$$

(Ii) Cycle of length 16:

$$v_1v_{22}v_{32}v_{42}v_{52}v_{62}v_{72}v_{82}v_9v_{83}v_{73}v_{63}v_{53}v_{43}v_{33}v_{23}v_{13}v_1$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$v_1v_{2(2m)}v_{3(2m)}v_{4(2m)}v_{5(2m)}v_{6(2m)}v_{7(2m)}v_{8(2m)}v_9v_{8(2m+1)}v_{7(2m+1)}v_{6(2m+1)}v_{5(2m+1)}v_{4(2m+1)}v_{3(2m+1)}v_{2(2m+1)}v_{1(2m+1)}v_1$$

It has been proved in [3] that cycles of length 9 and 16 are nullset magic. Therefore octagonal books with odd number of pages are nullset magic decomposable.

CONCLUSION

The main work carried out in this paper is the introduction of new types of decomposition. This will prove to be fruitful if it helps to widen the applicationarena of graph theory.

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